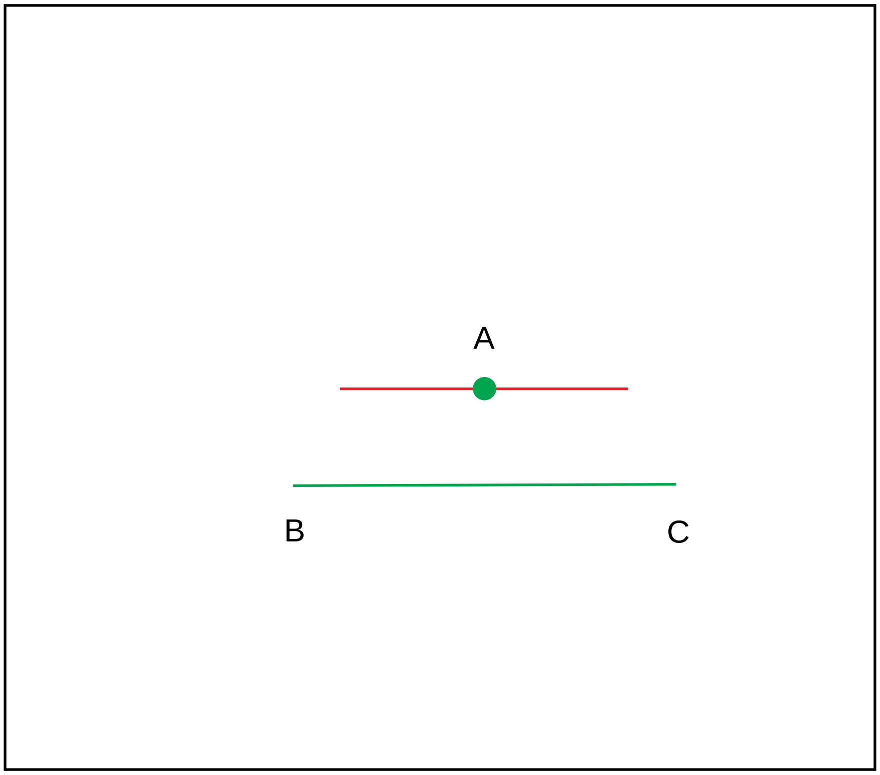
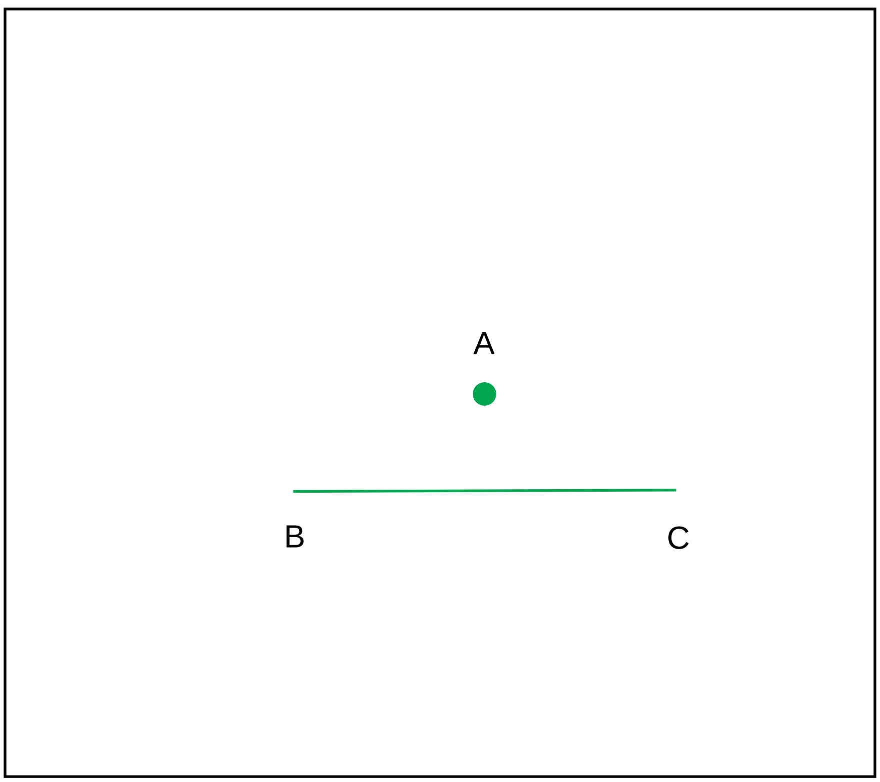
Construction 10: Book I, Proposition 31

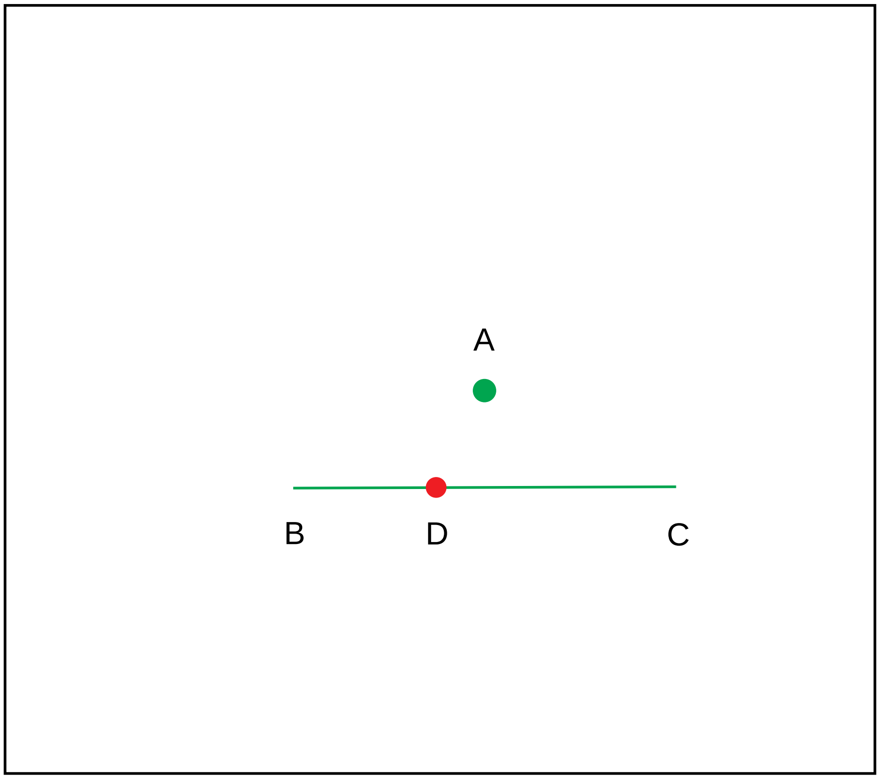
Through a given point to draw a straight line parallel to a given straight line.



I.31:3. Let A be the given point, and BC the given straight line.

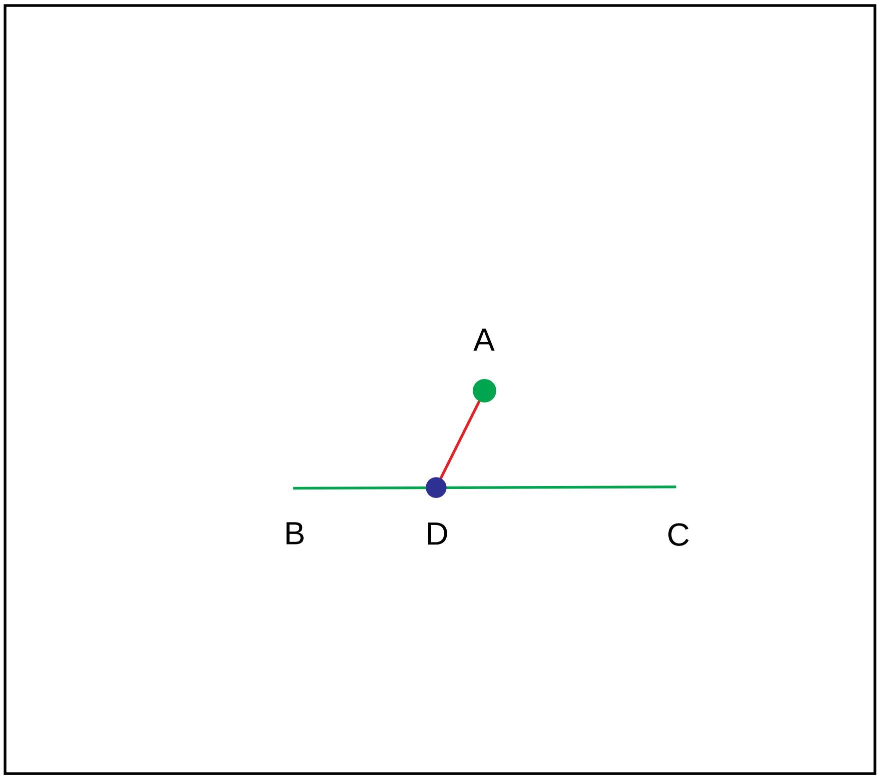


I.37:7. Let a point D be taken at random on BC,

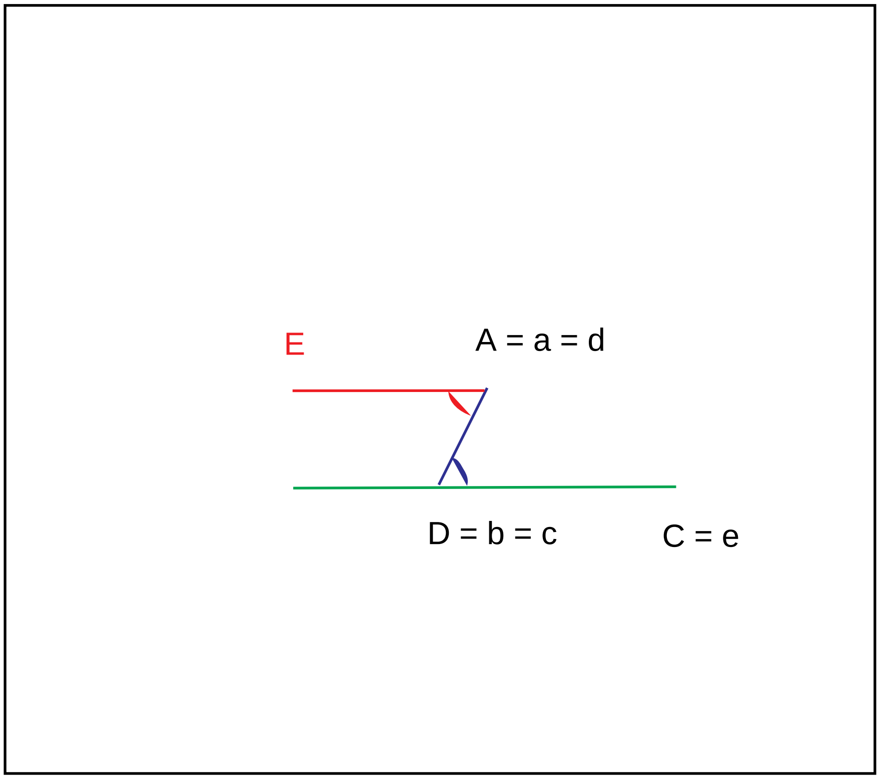


Note: Here we could take D=B to simplify the construction, but following Euclid, we regard D as an approximation to the point on BC closest to A. This improves the accuracy of the construction.

and let AD be joined;



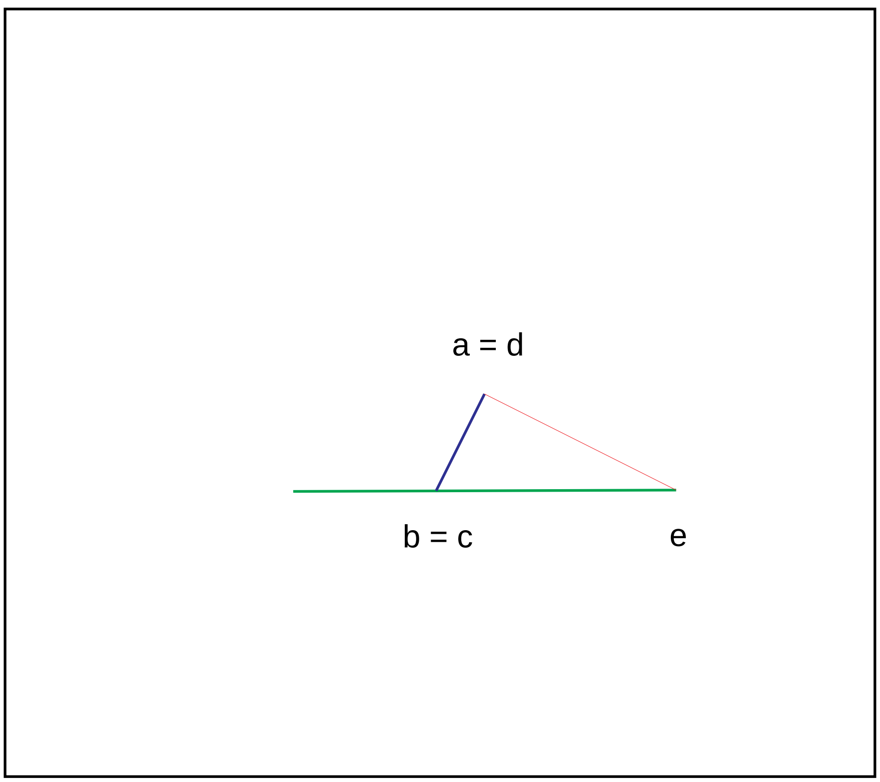
I.31:8. on the straight line DA, and at the point A on it, let the angle DAE be constructed equal to the angle ADC [I.23];
GOSUB I.23



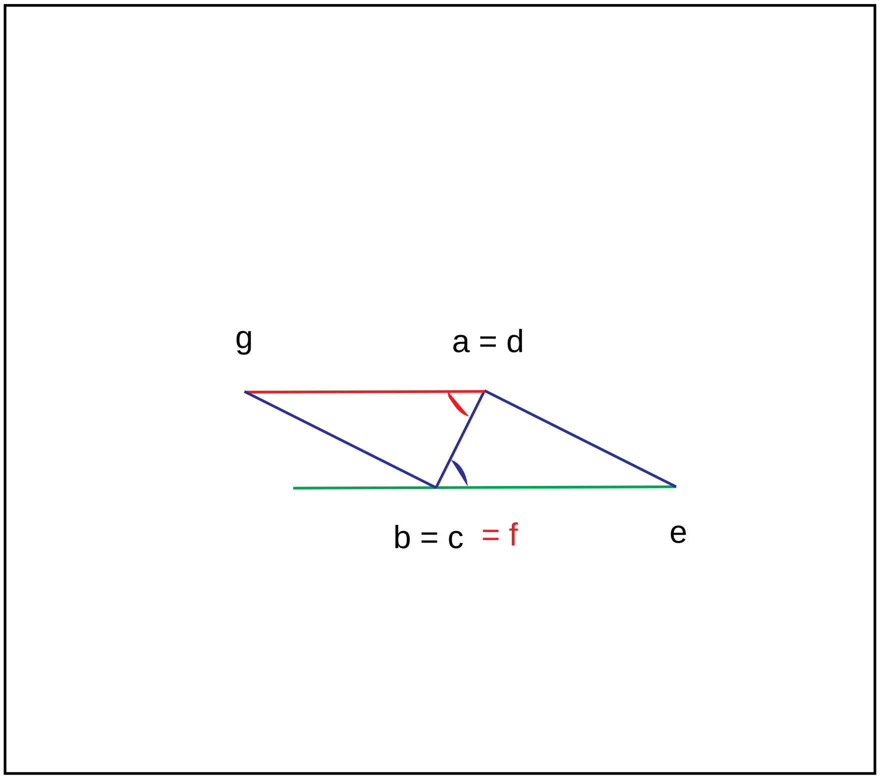
Relabel.

Note re. I.23. The points on the straight lines enclosing the angle dce are already given, and are the points d and e. Also, one of these enclosing lines coincides with the the target straight line, ab, to which the angle is to be moved. Hence we will follow a very special case of I.23. We join I.23 at line 10.

I.23:10. let de be joined,



I.23:11. and out of three straight lines which are equal to the three straight lines cd, de, ce let the triangle afg be constructed in such a way that cd is equal to af, ce to ag, and further de to fg. [I.22]
GOSUB I.22.

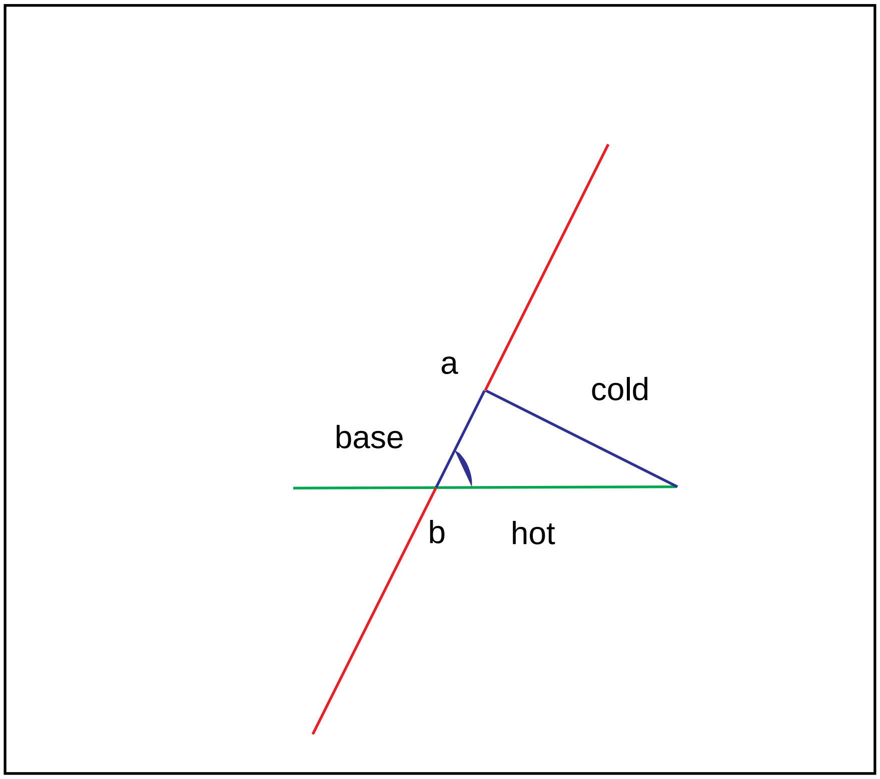


We have shown here just one of the ways that I.22 may be fitted to this phrase of Euclid. We may think of the triangle rotating rigidly around the midpoint of cd.

Note: Actually we want I.22P here, as in C#9. Further, as we are constructing a rather special case of I.23, we need a special case of C#8P = I.22P. We will use the form given above in terms of the base, the hot arm, and the cold arm.

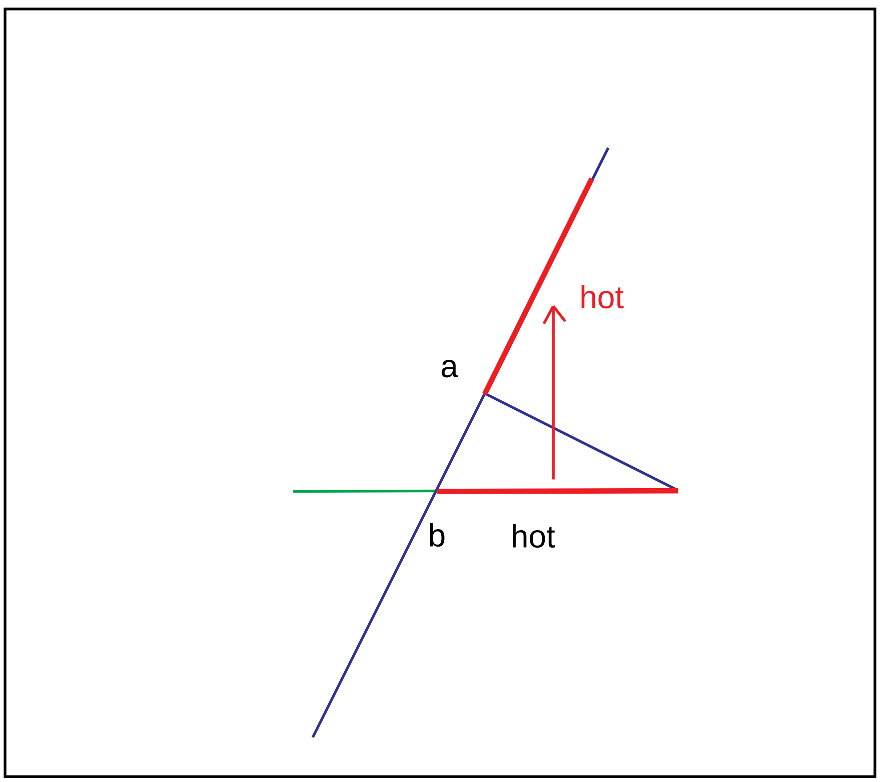
We are to “move” the triangle cde to the target straight line ab so that ce moves onto ab. Regard cd as the base(we could alternatively choose ce as base.) As its hot endpoint c is to fall on the hot endpoint a of ab (note before moving that b=c, that is the hot end of ce is now at the cold end of ab), the hot arm of the triangle cde is ce. Now we follow I.22P.

I.22P: Interpolated step. Extend the straight line ab.

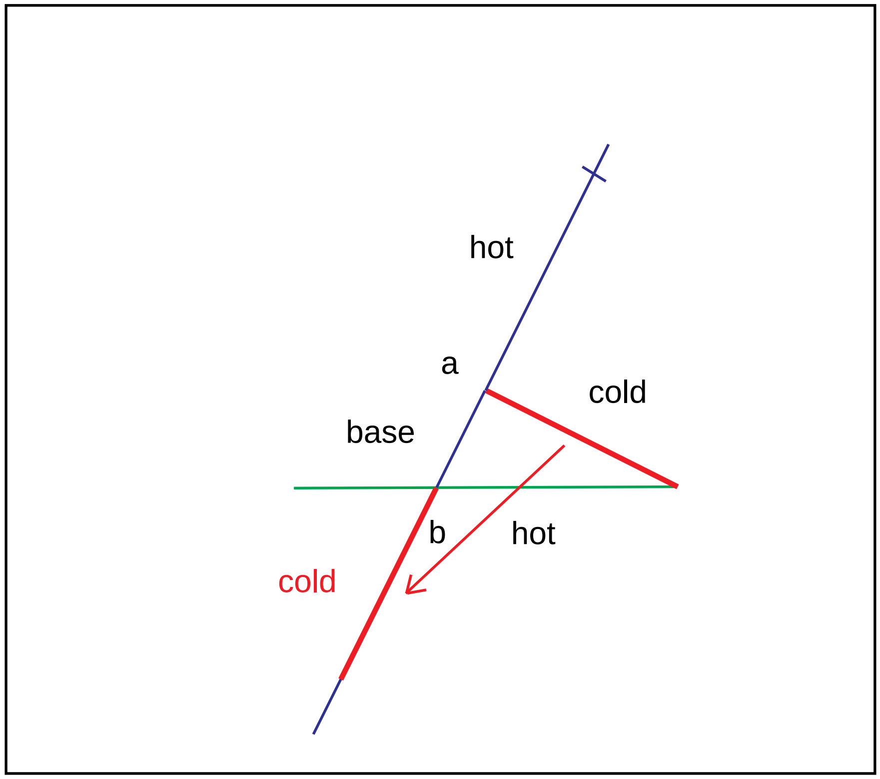


Note: In this special case of I.22P, we are already given the endpoint of the moved base. It is b, as ab = dc. The hotpoint is a.

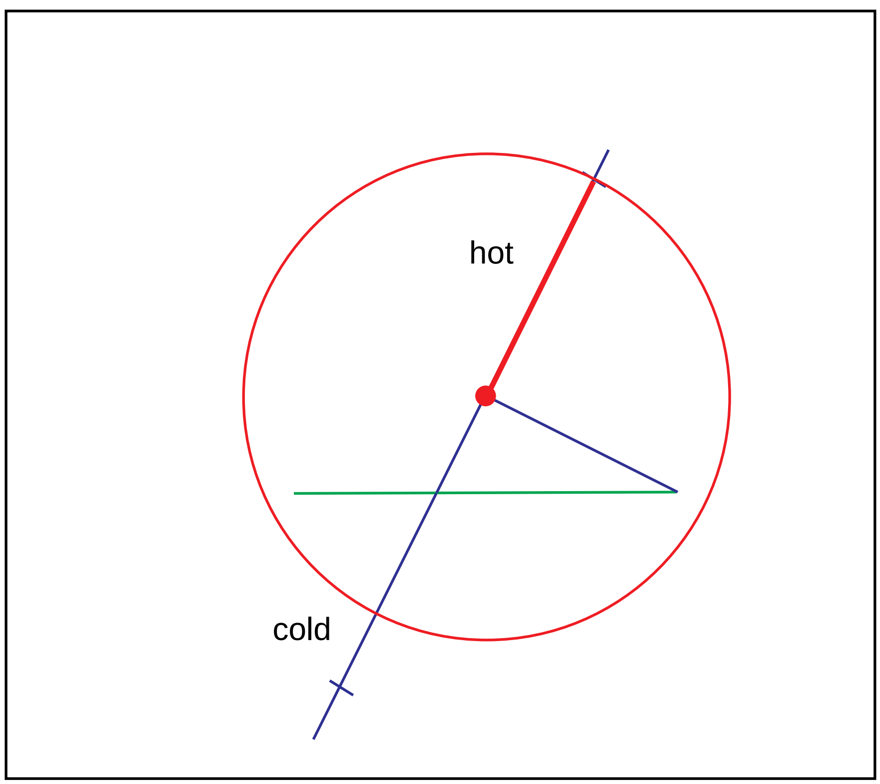
We paraphrase I.22P to avoid a confusion of labels. Move the hot arm of the triangle to the hot end of the extended line (dividers).



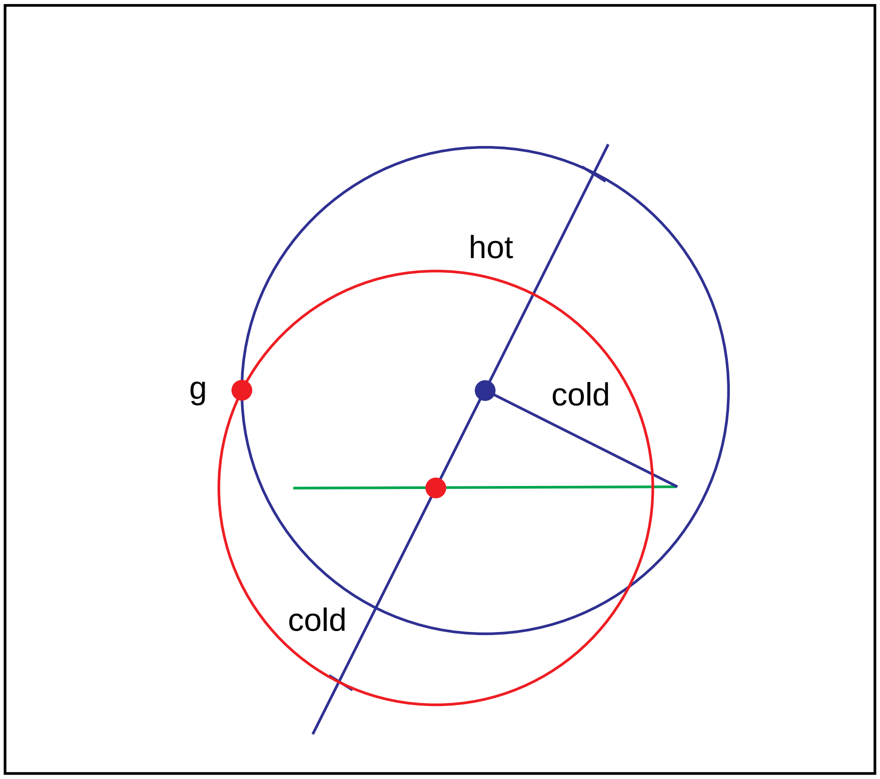
Move the cold arm of the triangle to the cold end of the extended line.



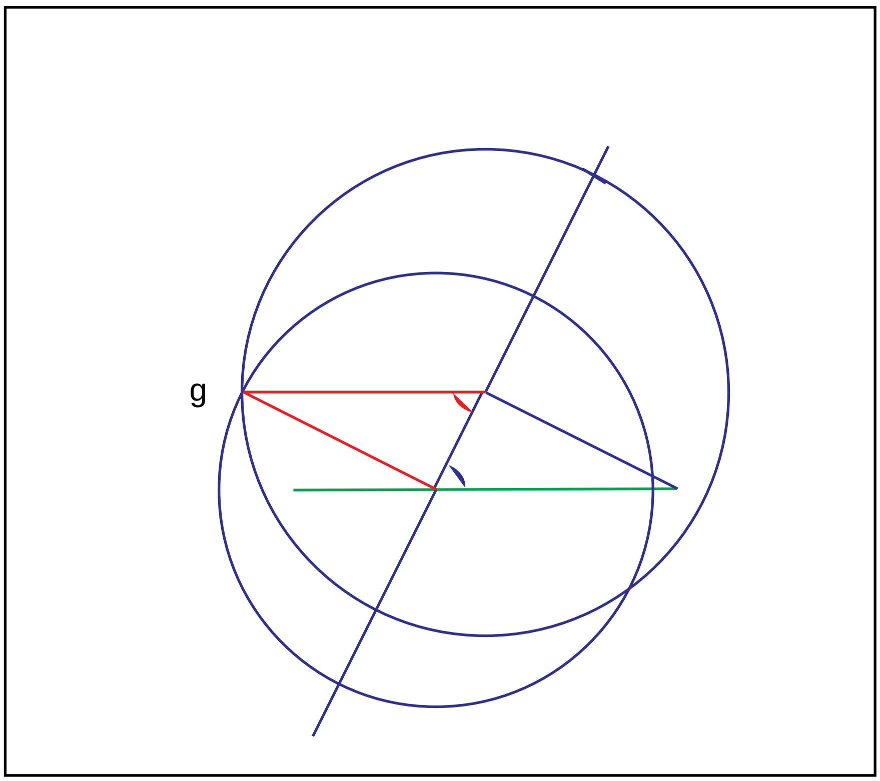
Swing the hot arm around the hot end of the base.



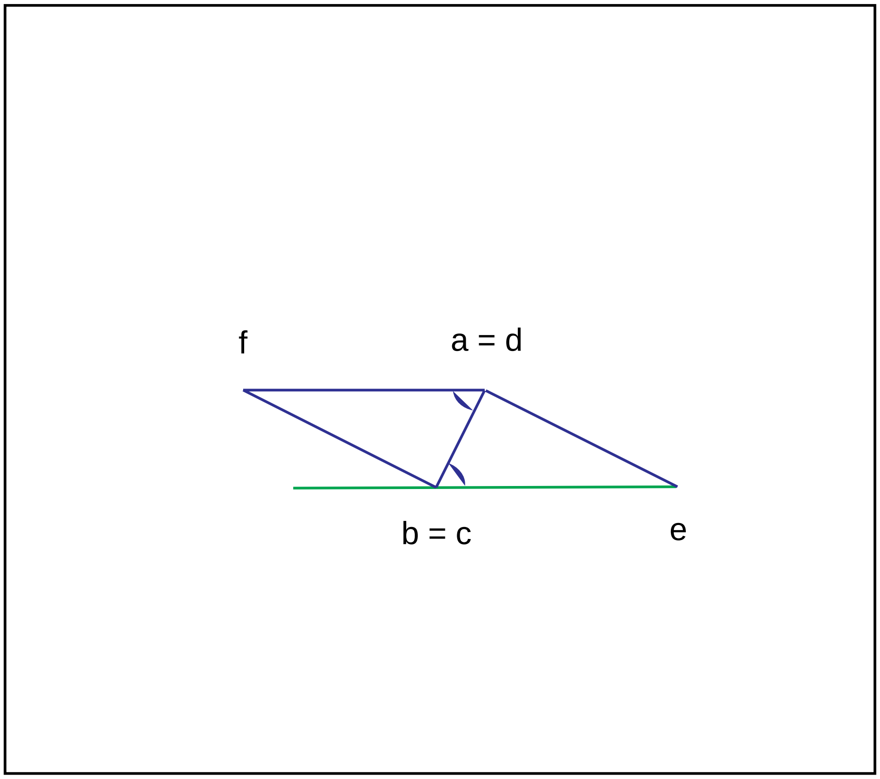
Swing the cold arm around the cold end of the base. Mark the point g where the two circles meet on the side of the extended line opposite to the triangle.



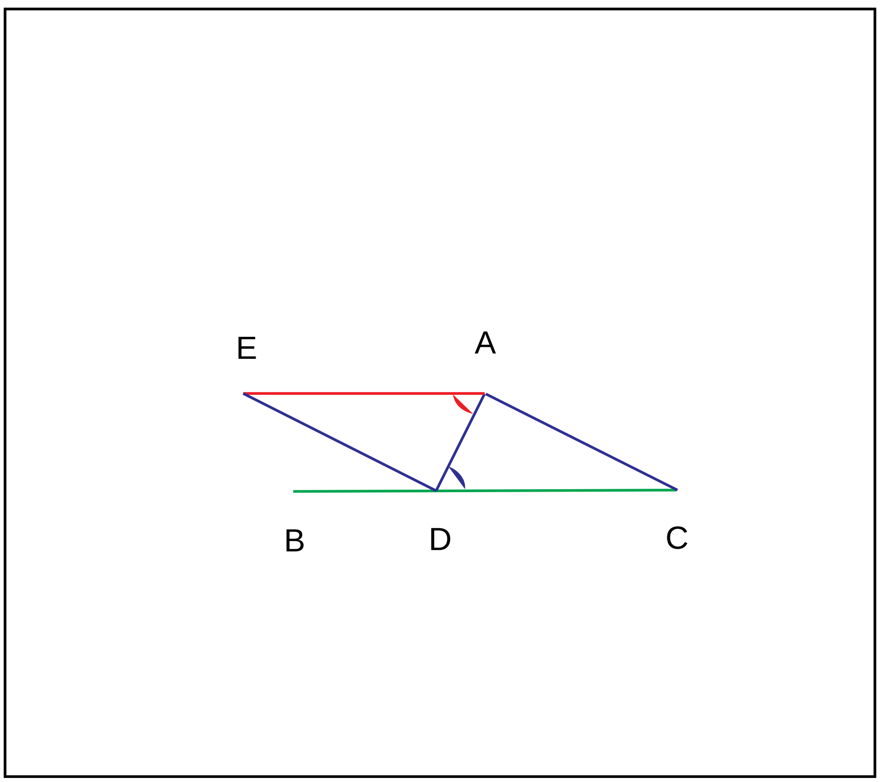
Connect g to both ends of the base.



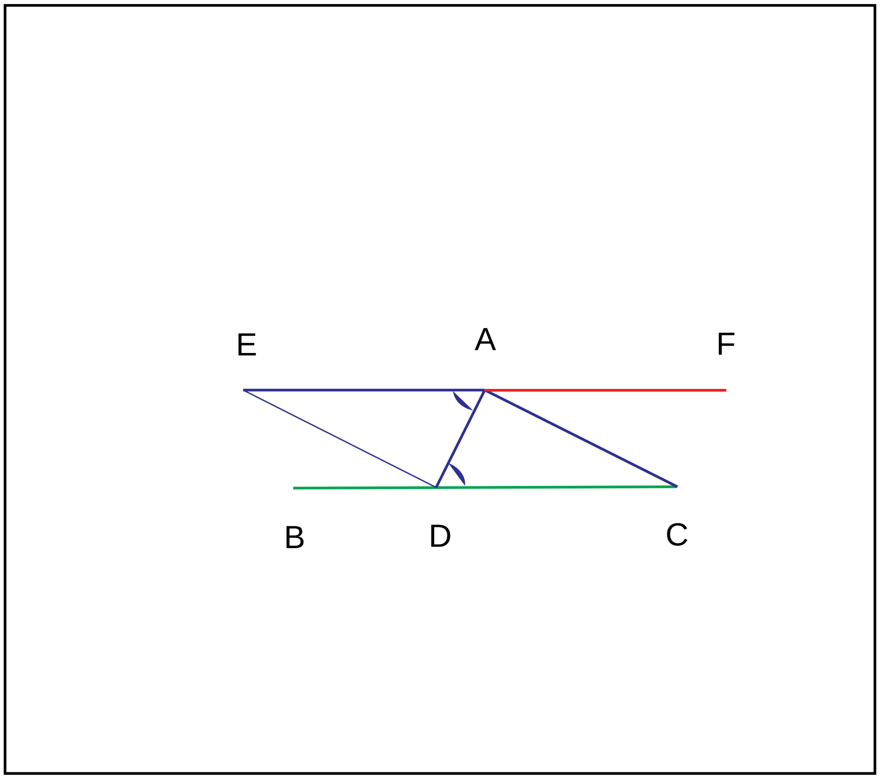
Cleanup and RETURN from SUB I.22P to I.23 at line 11.



Cleanup and RETURN from SUB I.23 to I.31 at line 8.



I.31:11. and let the line AF be produced in a straight line with EA.



I.37:18. Therefore through the given point A the straight line EAF has been drawn parallel to the given straight line BC.

Q.E.F.