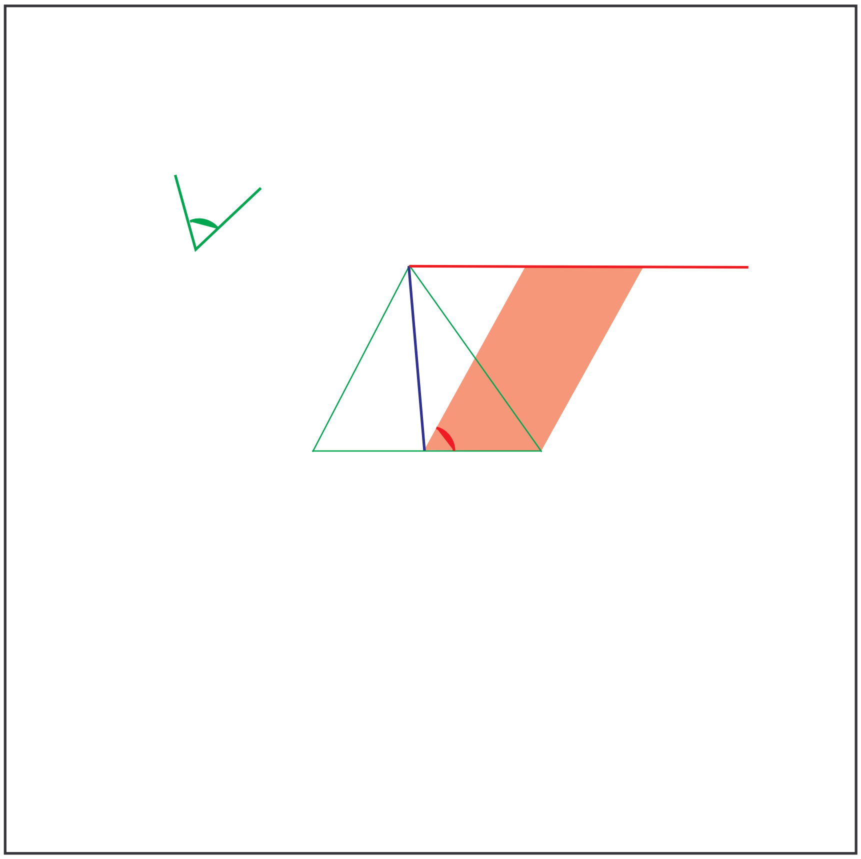
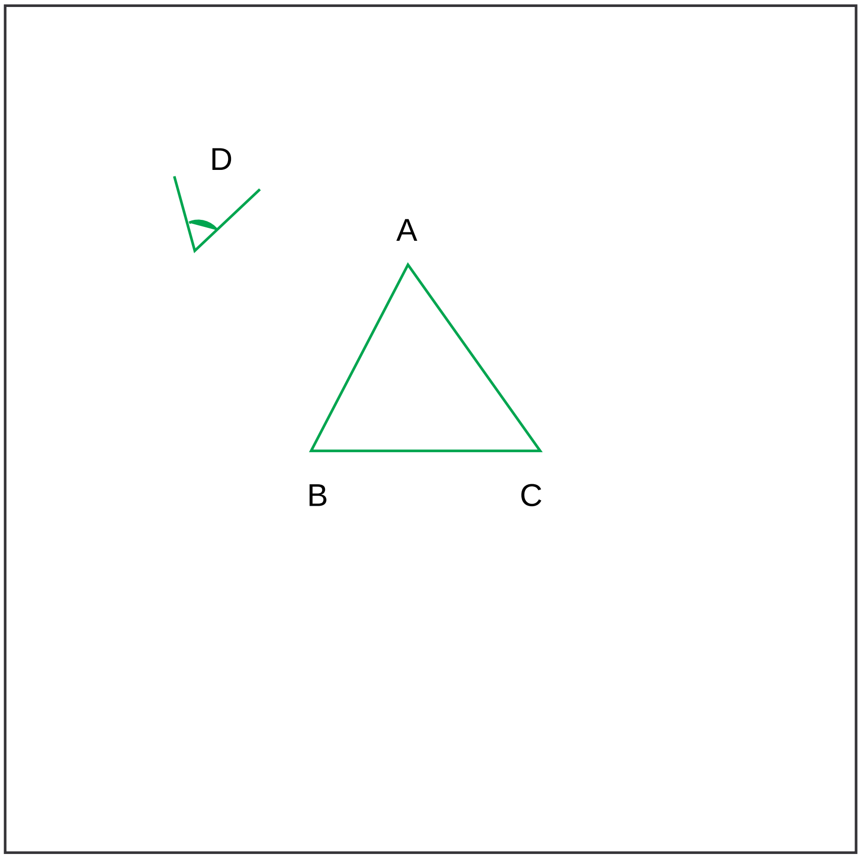
Construction 11: Book I, Proposition 42

To construct, in a given rectilineal angle, a parallelogram equal to a given triangle.

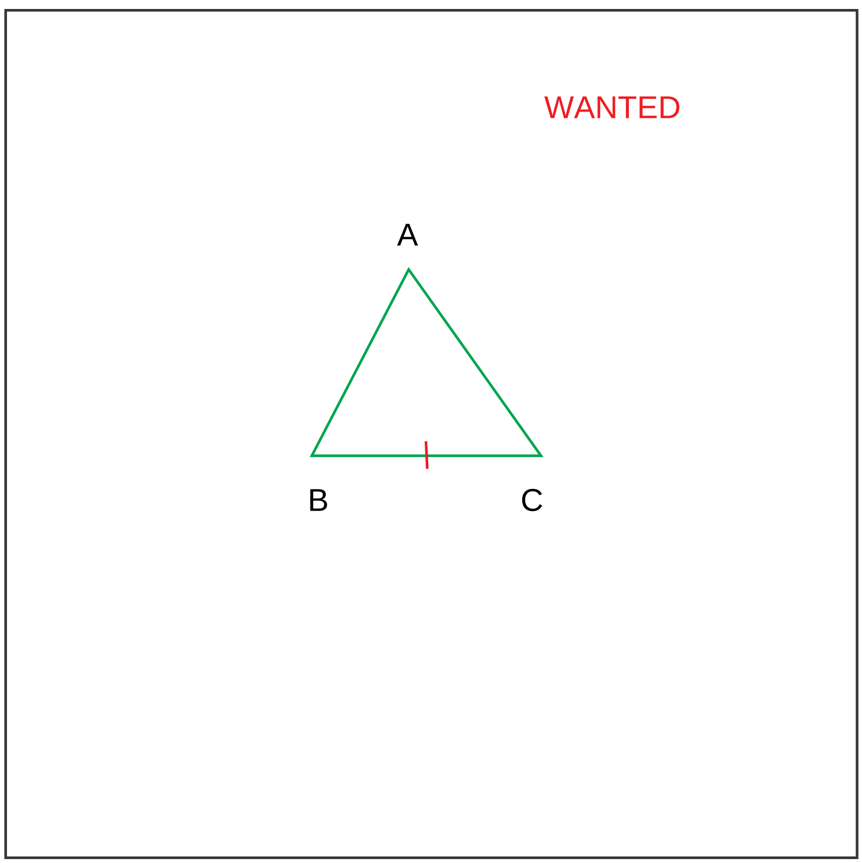
Note: Equal here means equal in area.



I.42:3. Let ABC be the given triangle, and D the given rectilineal angle;



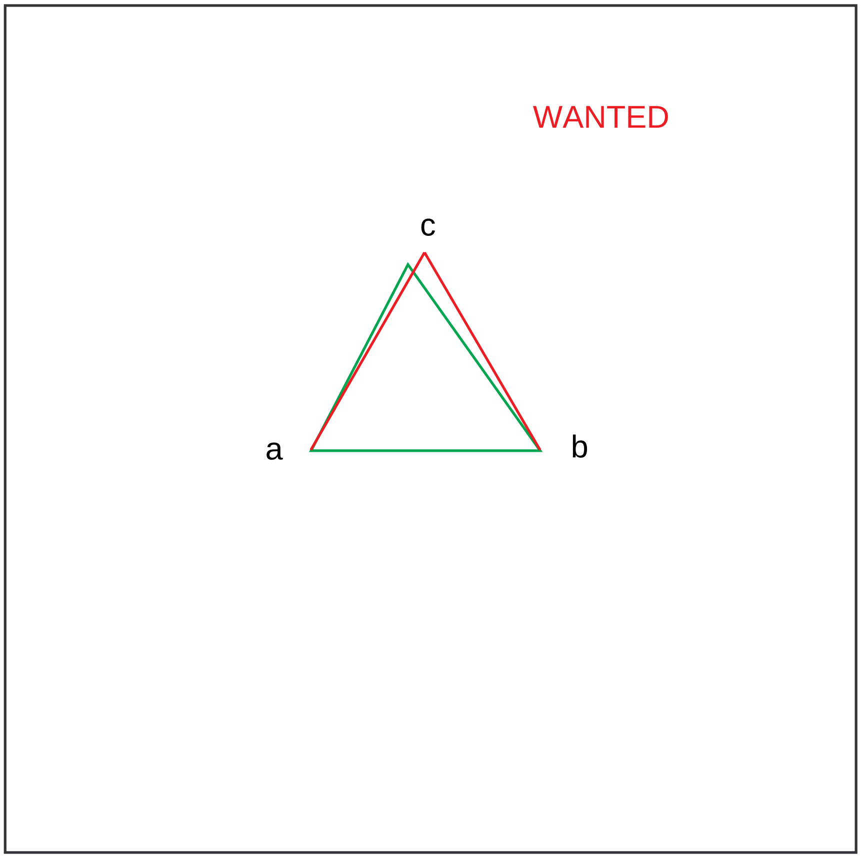
I.42:8. Let BC be bisected at E, ([I.10])



Relabel.

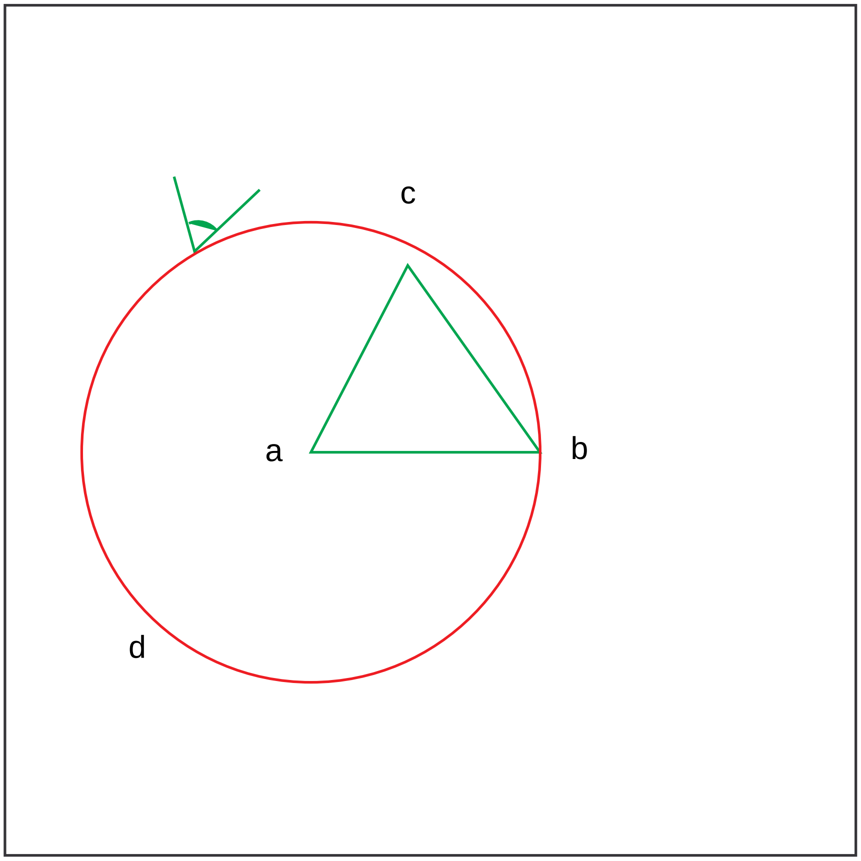
GOSUB I.10.4

Let the equilateral triangle abc be construted on it, [I.1]

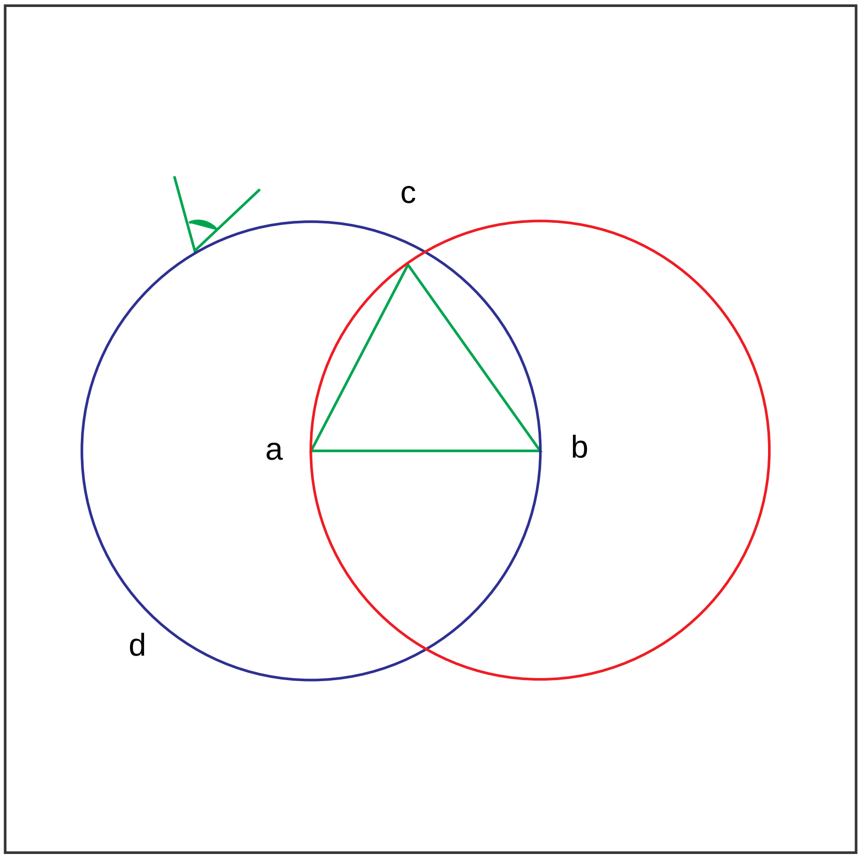


GOSUB I.1.

I.1:7. With centre a and distance ab let the circle bcd be described; [Post. 3]



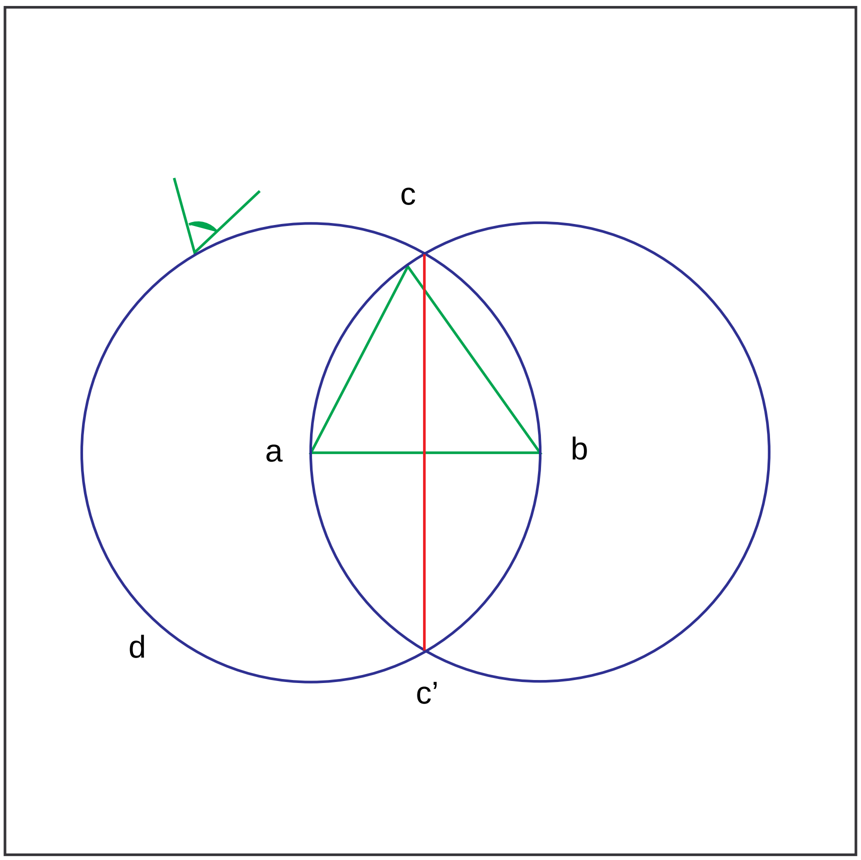
I.1:10. again, with centre b and distance ba let the circle ace be described;
[Post. 3]



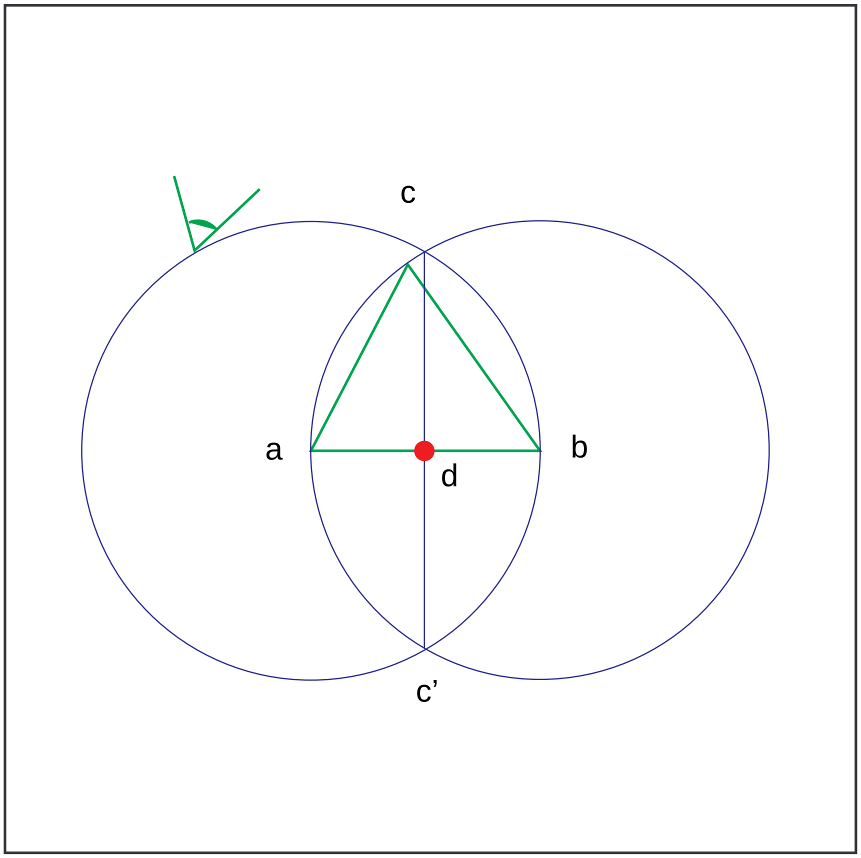
Note: In C#5 (I.10) there are several inessential steps at this point. Now we omit them.

RETURN to I.10 at line 4.

let cc’ be joined.



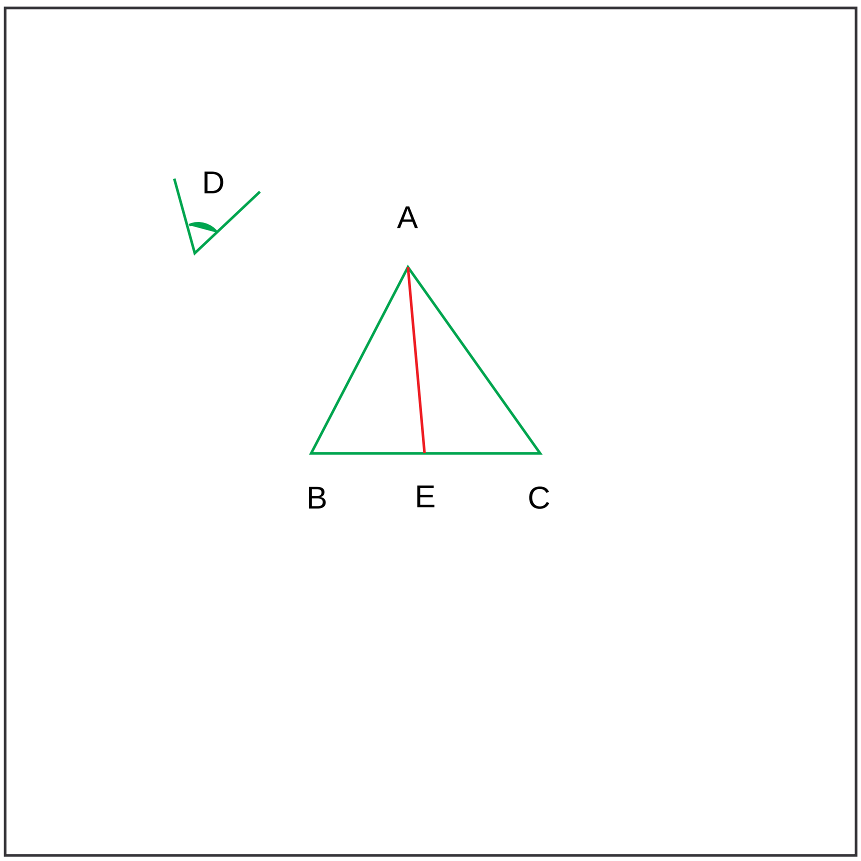
I.10:8. The straight line ab has been bisected at the point d.



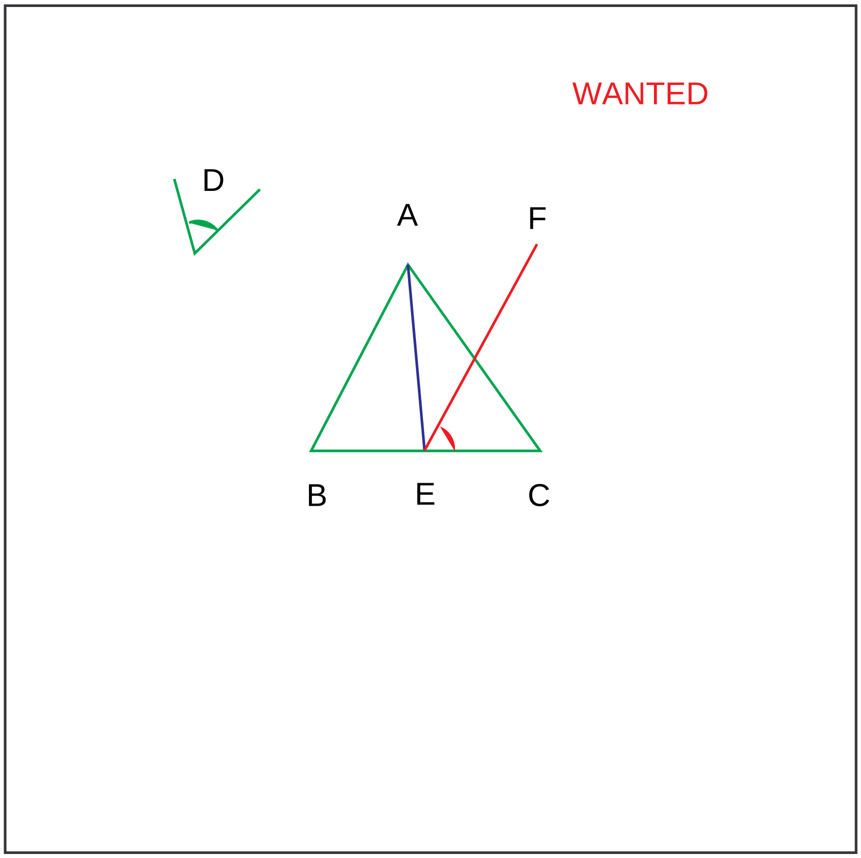
RETURN to I.42 at line 9.

Relabel, cleanup.

I.42:9. and let AE be joined, [Actually, this line will not be used.]

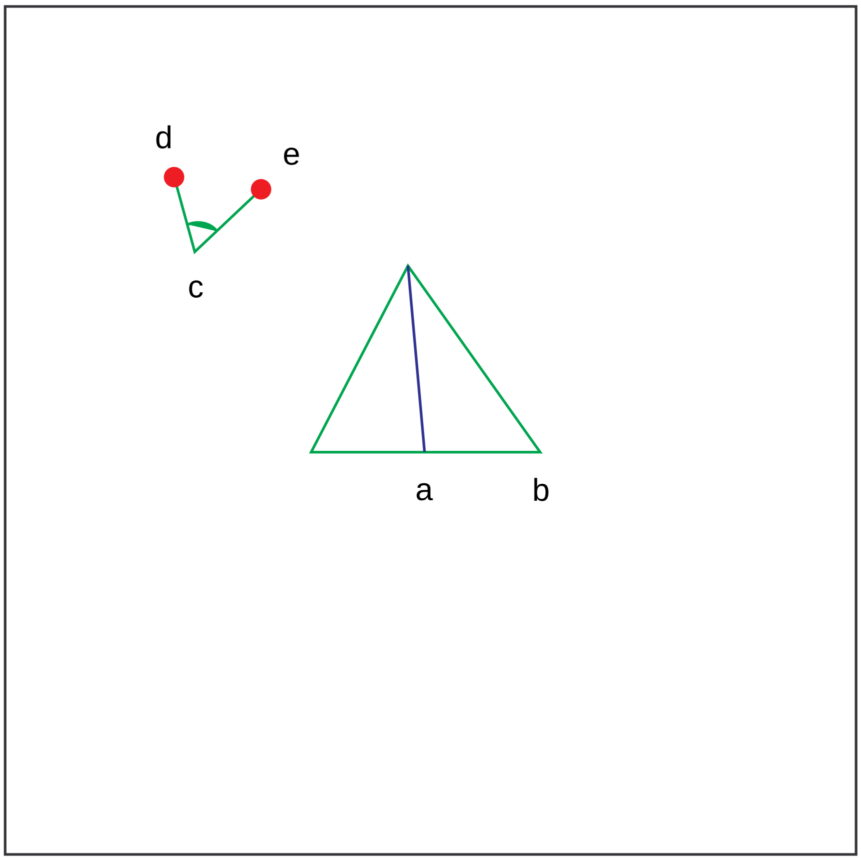


I.42:10. on the straight line EC, and at the point E on it, let the angle CEF be constructed equal to the angle D; [I.23]

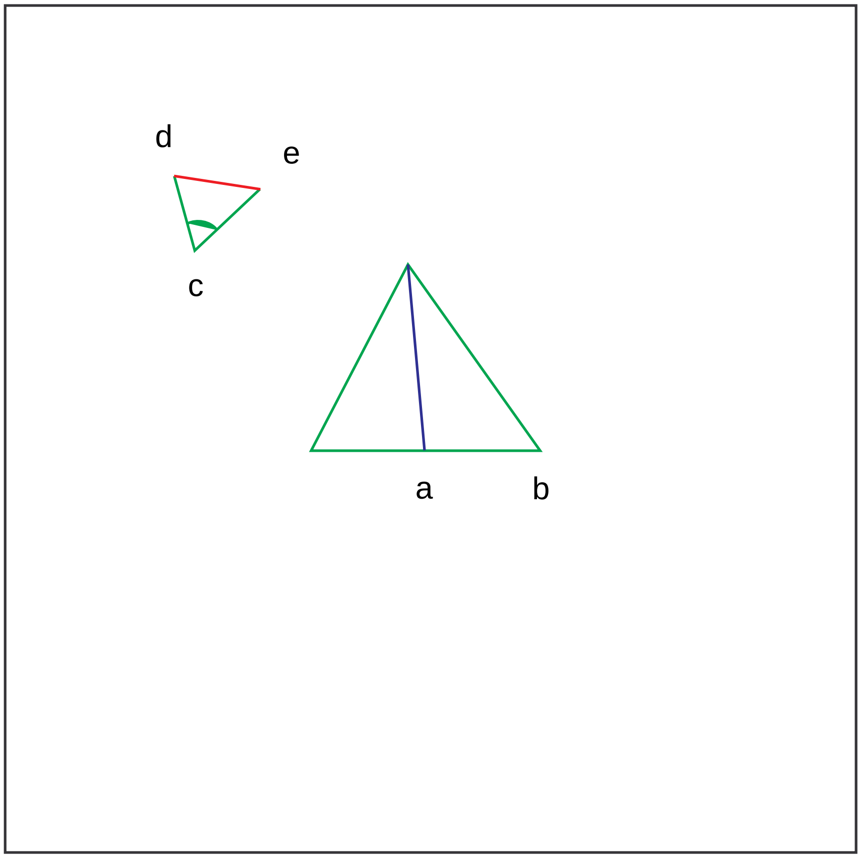


GOSUB I.23
Relabel.

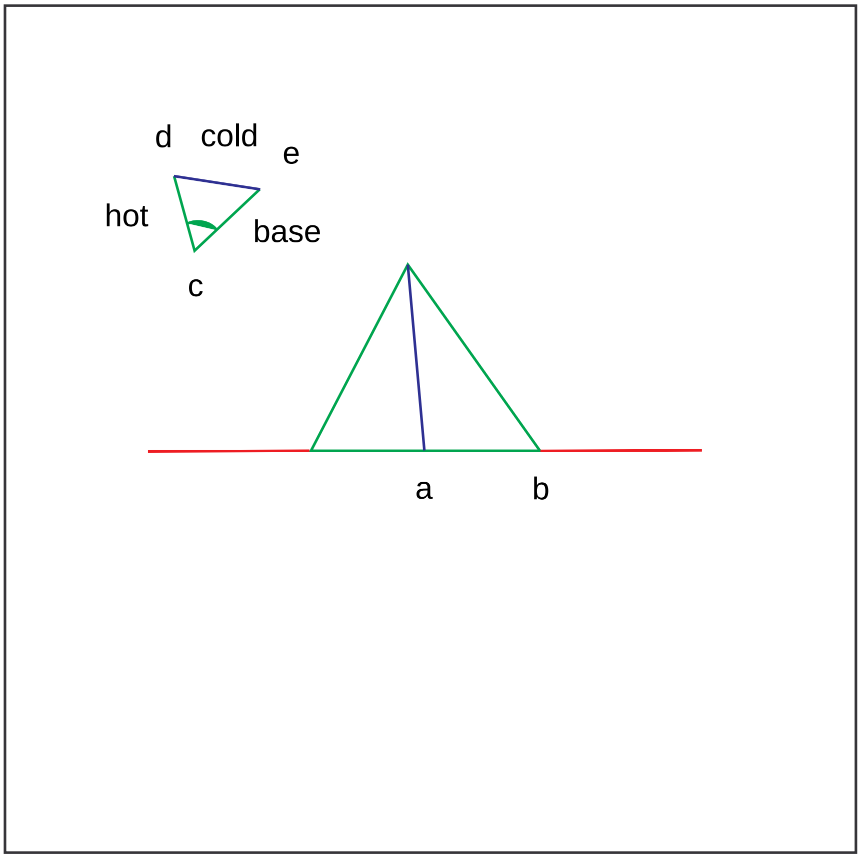
I.23:8. On the straight lines cd, ce respectively let the points d, e be taken at random;



I.23:10. let de be joined,

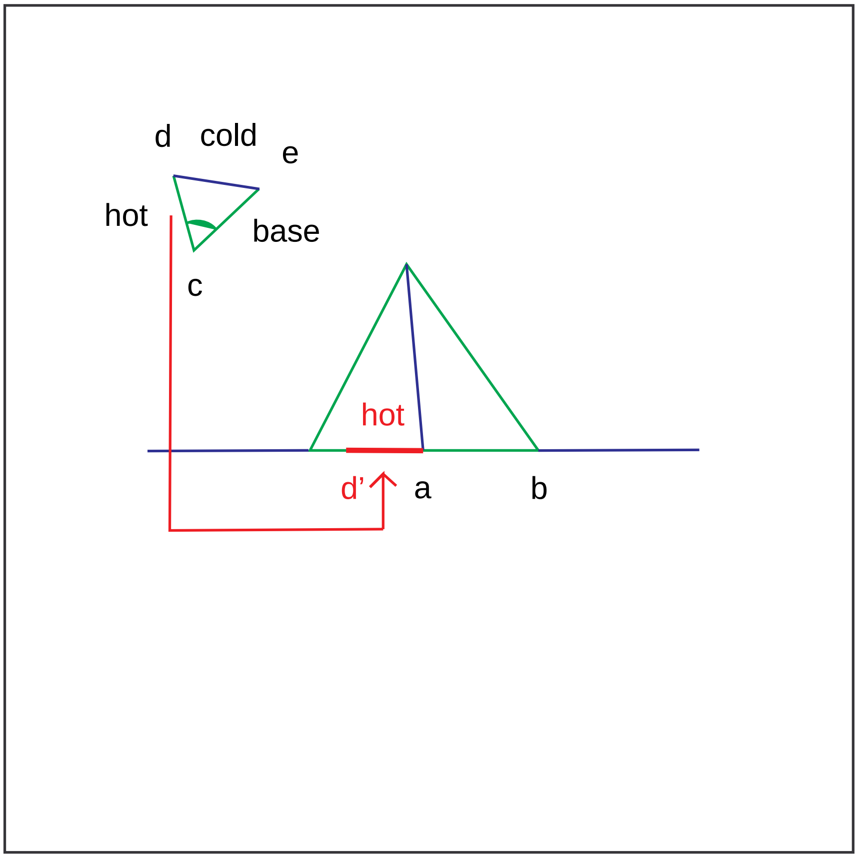


I.23:11. (Paraphrase) Move the triangle cde so the base ce goes to the line ab with the angle at c at the hot end, a.

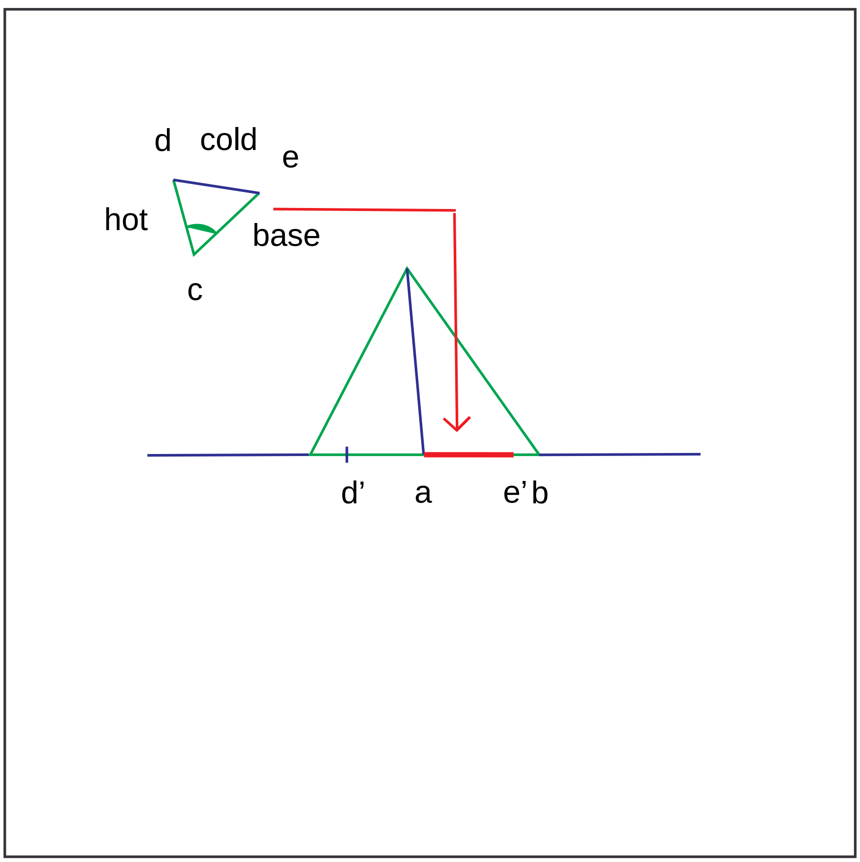


GOSUB I.22P.
We follow the summary. Extend the line ab.

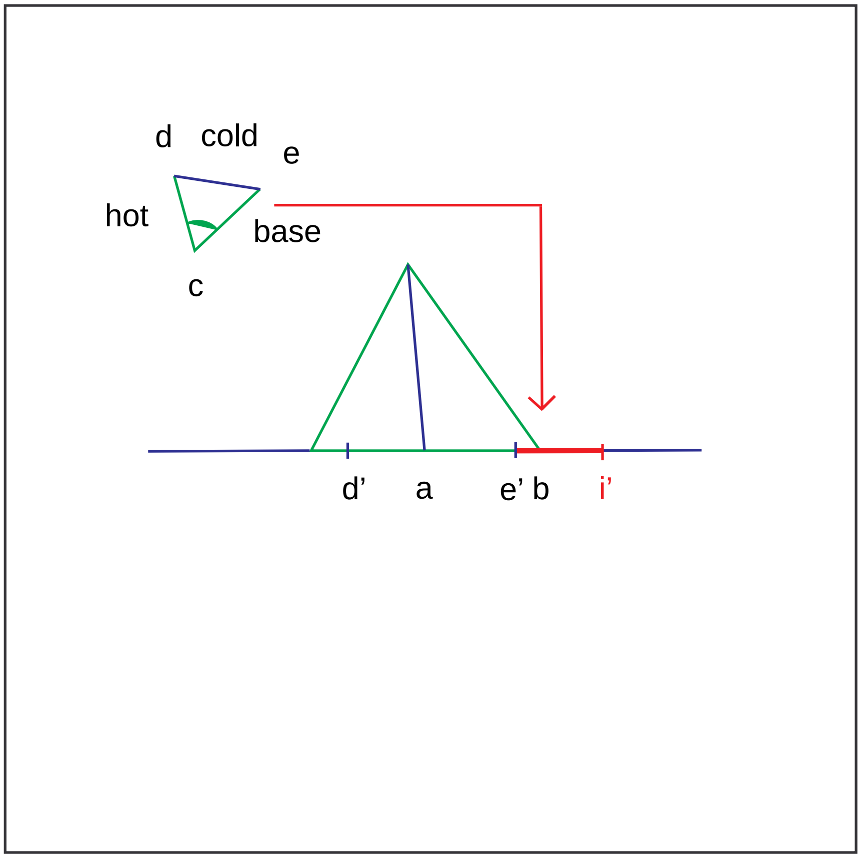
(Move the hot arm.) Locate the point d' so that ad' is equal to cd. ([dividers])



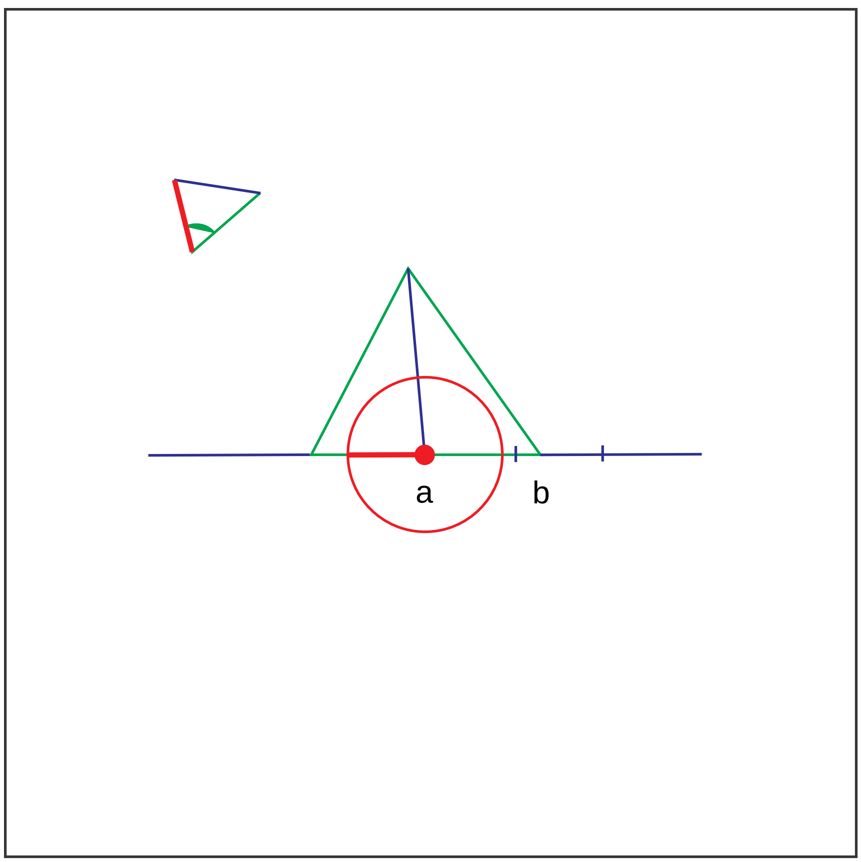
(Move the base.) Locate the point e' so that ae' is equal to ce. ([Dividers])



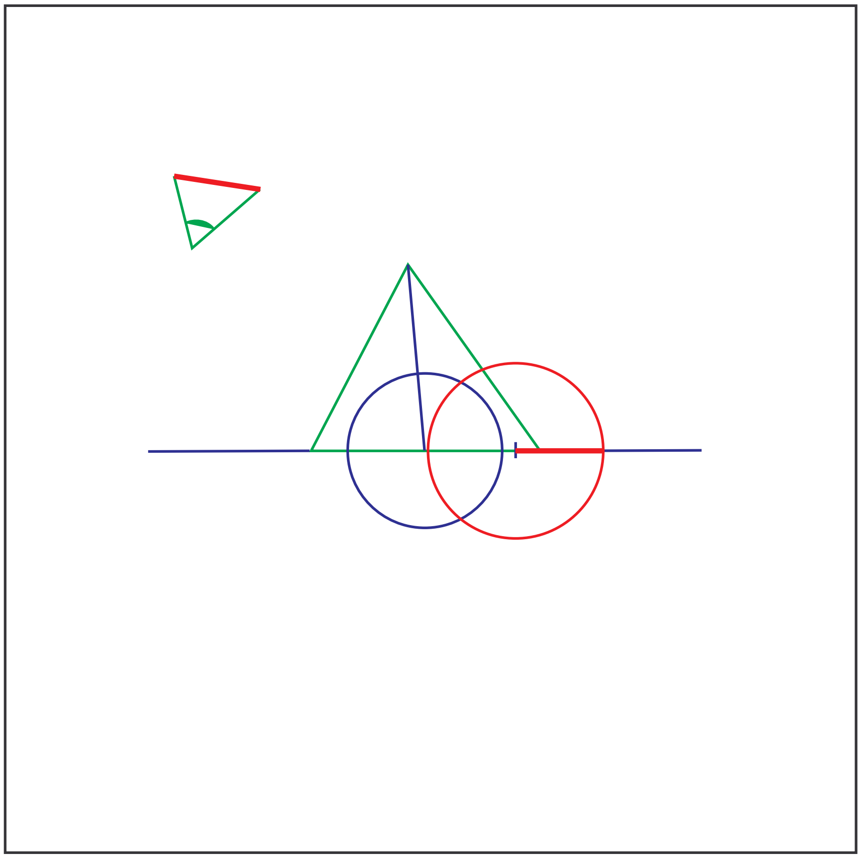
(Move the cold arm.) Locate the point i’ so that e’i’ is equal to de. ([Dividers])



I.22:24. Swing the hot arm.



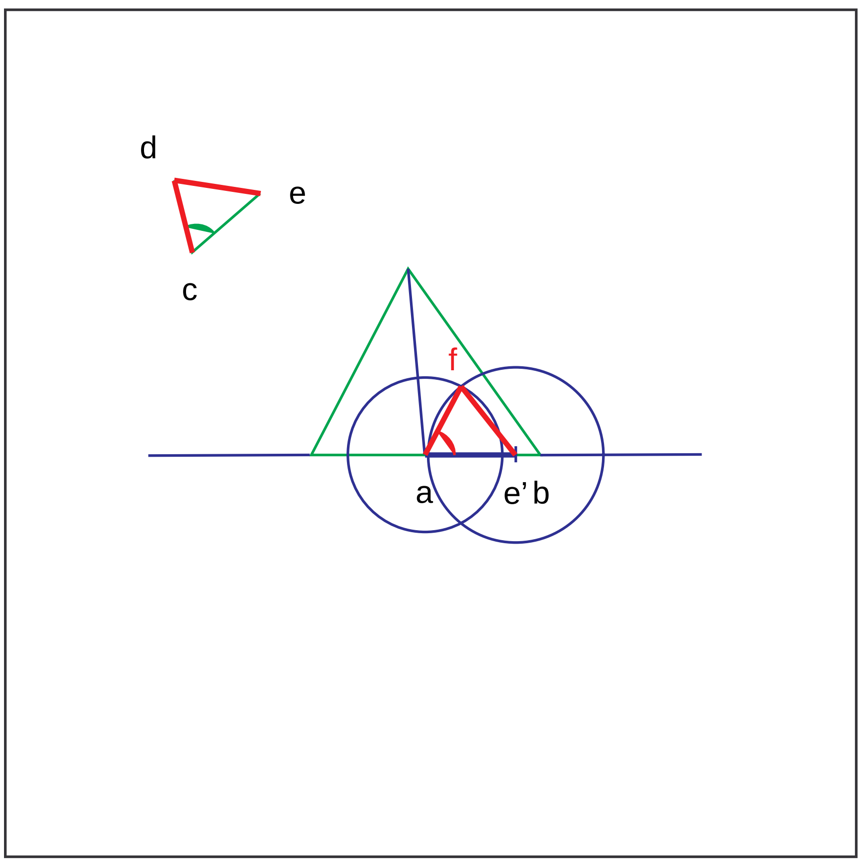
I.22:26. Swing the cold arm.



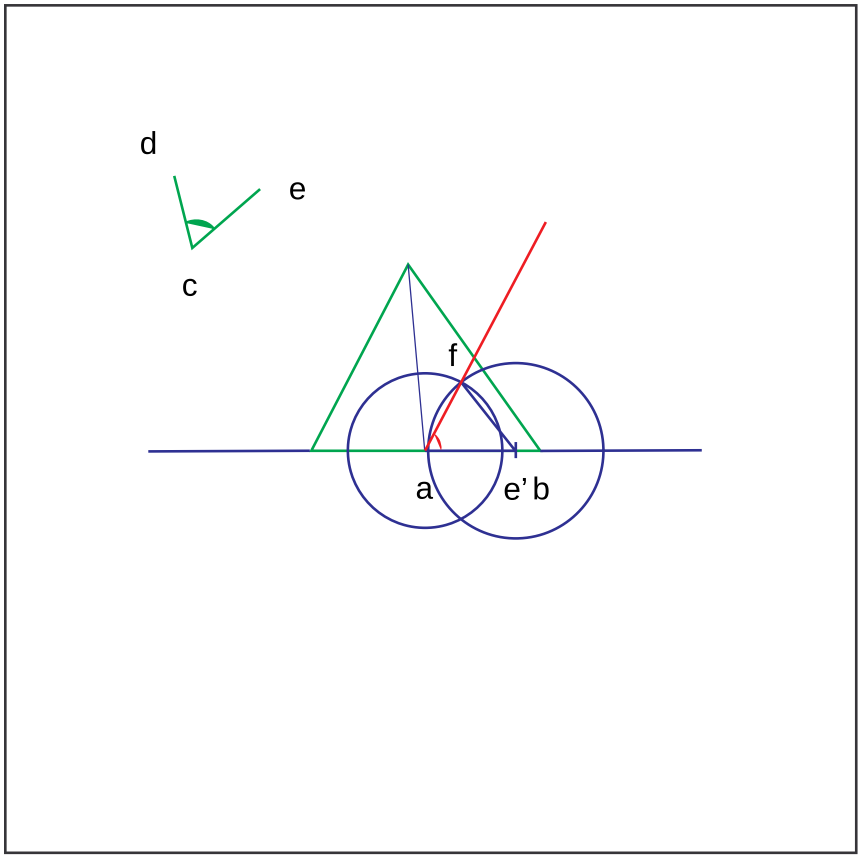
I.22:28. Join the point where the two arms meet, f, to the ends of the (moved) base.

RETURN to I.23 at line 11.

I.23:18. The angle dce is equal to the angle fae'. RETURN to I.42 at line 10.



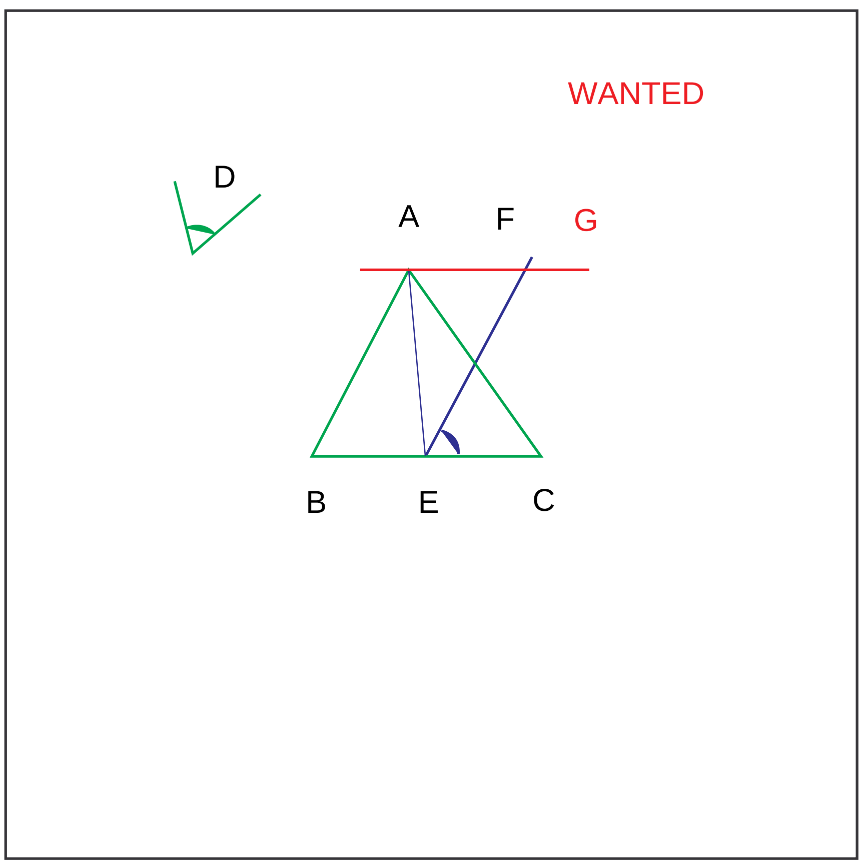
Extend the hot side, af.



Cleanu.

RETURN to I.42 at line10.

I.42:14. Through A let AG be drawn parallel to EC, [I.31]

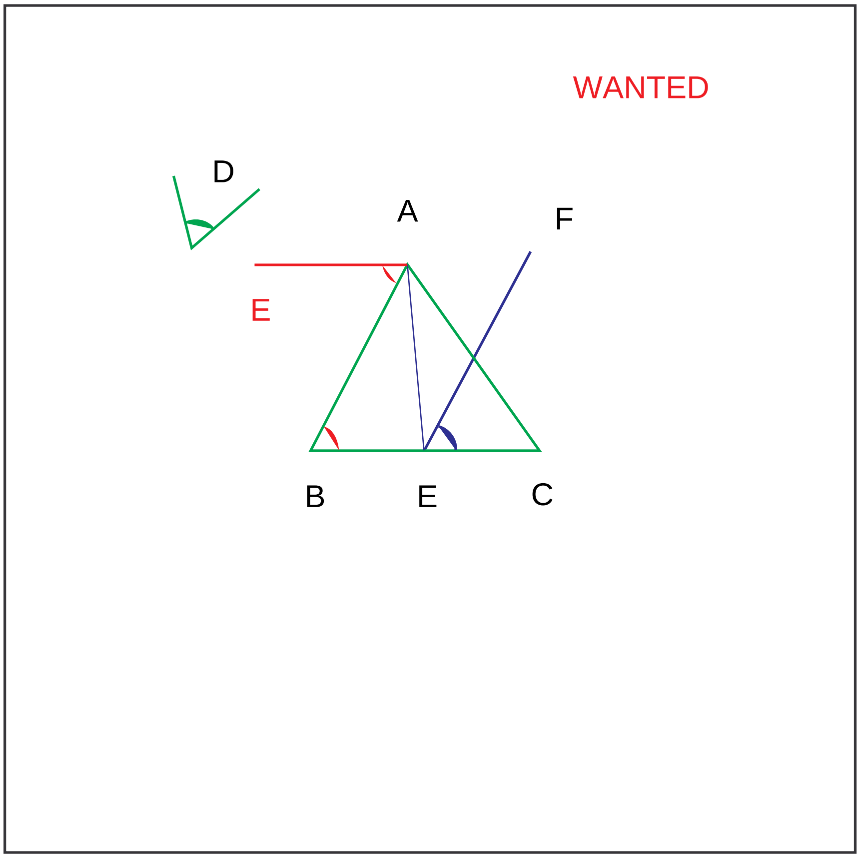


GOSUB I.31.
We will take BC as the given line, so it is not necessary to relabel. We take the point “D” of I.31 as B, as noted in C#10. A”D” = AB is already joined. We take up I.31 at line 8.

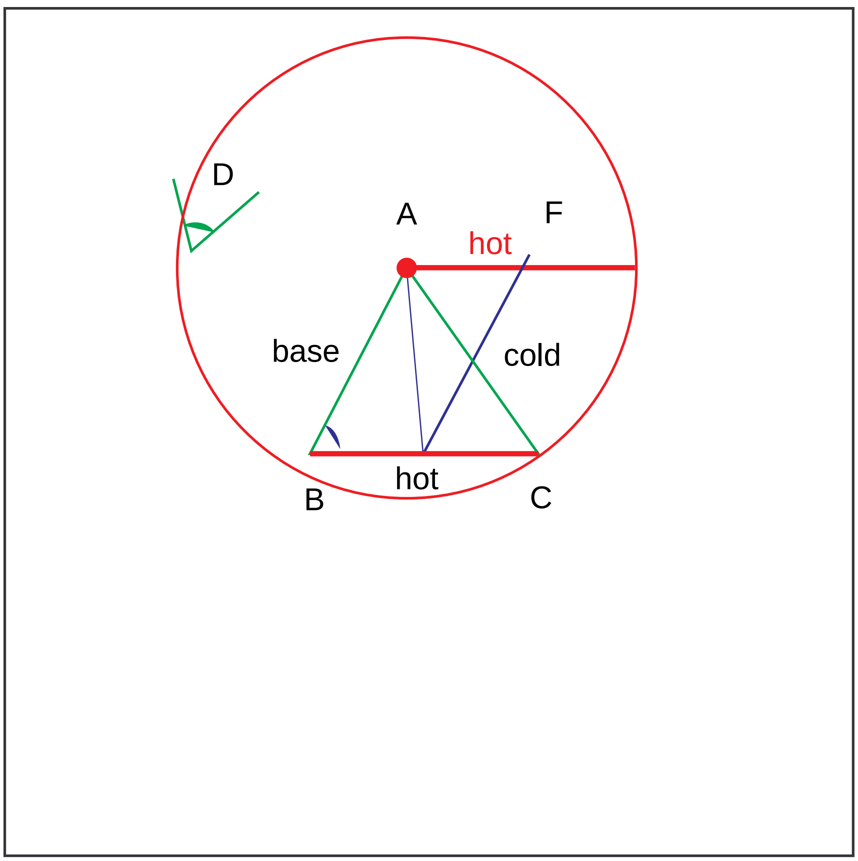
I.31:8. On the straight line BA and at the point A on it, let the angle be constructed equal to the angle ABC [I.23];

GOSUB I.23

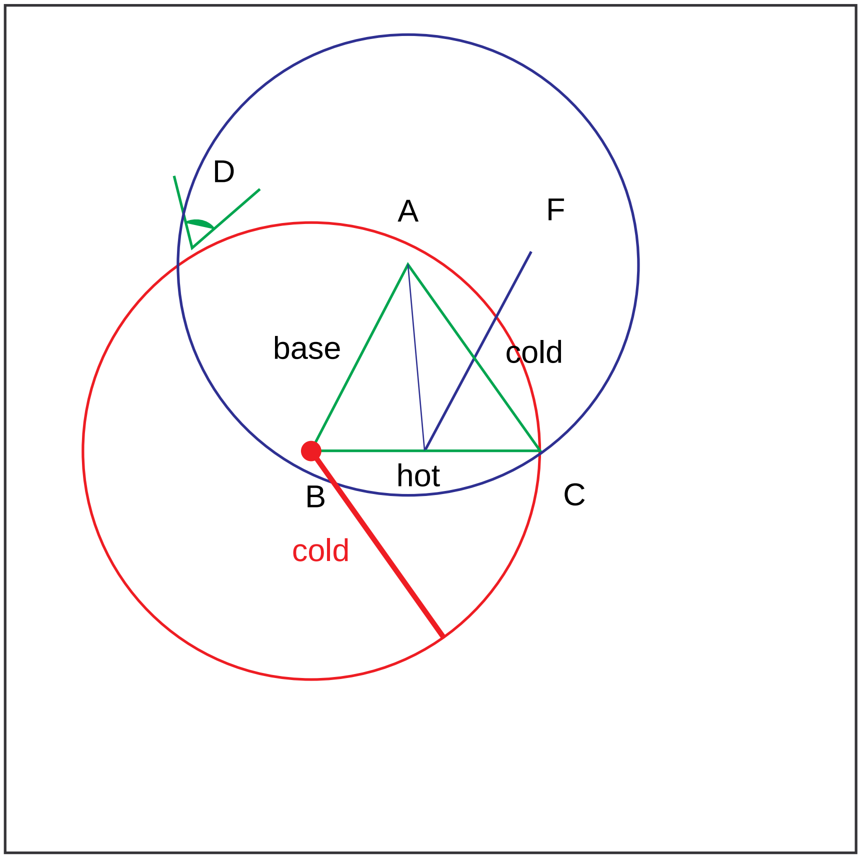
We must move a triangle enclosing the angle ABC, but ABC is already a triangle. Joining I.23 at line 12, which is where I.22 is called to move the triangle, but we must use the Proclus Variation, I.22P. As usual, we follow the summary, in terms of the base and the two arms, hot and cold. This time, we will further shorten that routine by swinging each arm as soon as it is moved. The interpolated step (extending the target line) is then unnecessary.



I.22P. Move the hot arm to the hot end A of the target line AB and swing it. (Compass, [Post. 3])

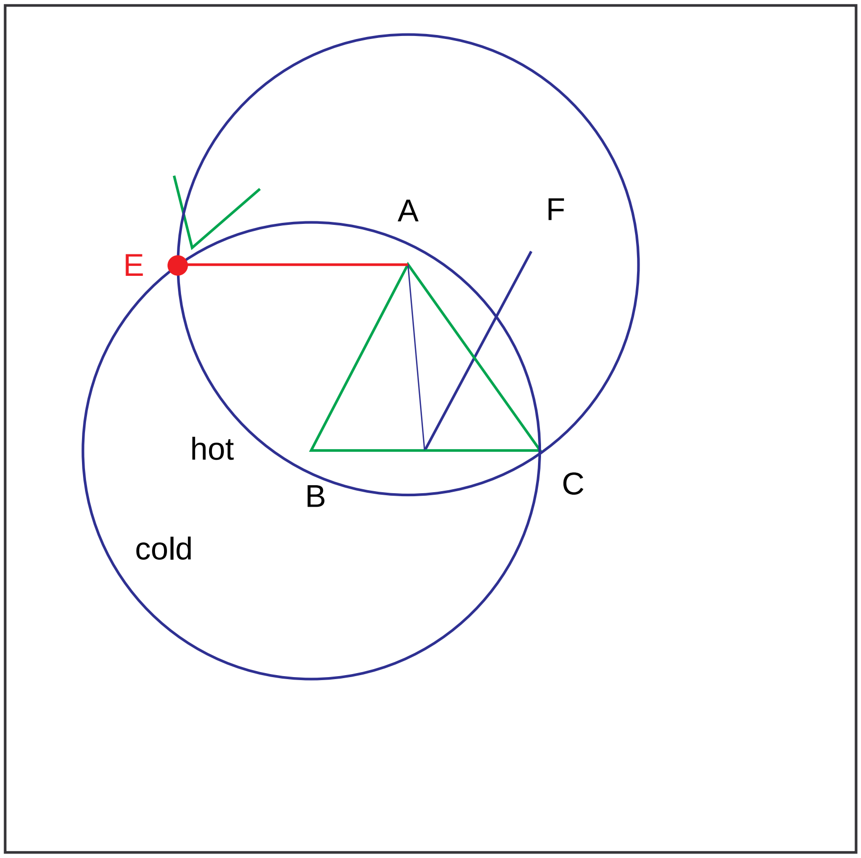


I.22P. Move the cold arm to the cold end B of the target line AB and swing it.

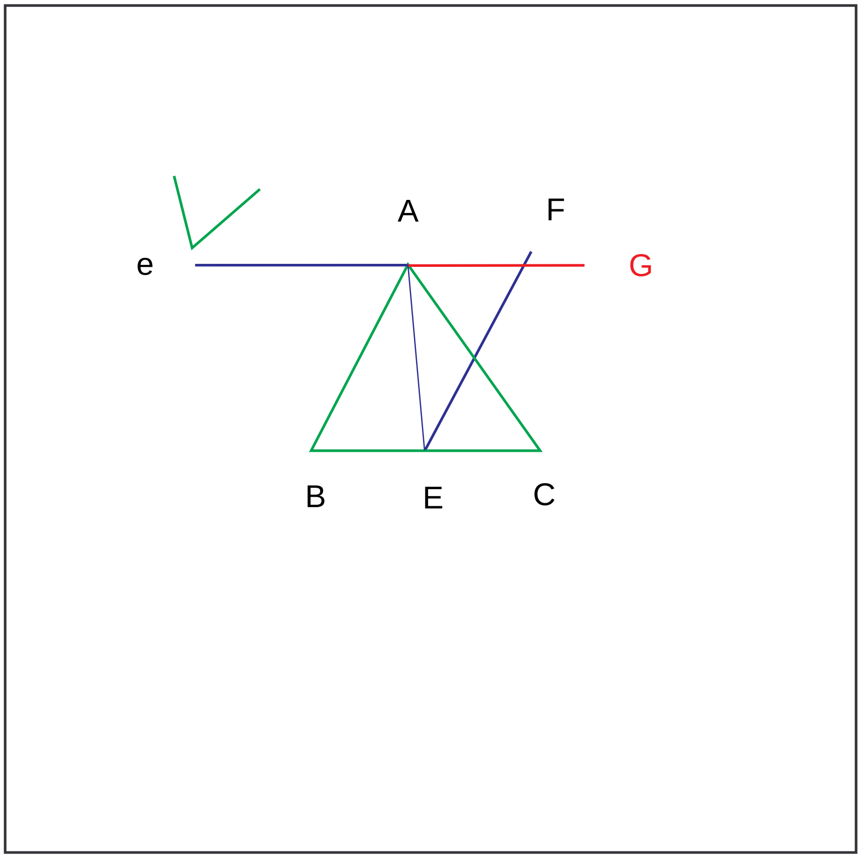


I.22P. Connect the meeting point E of the hot and cold circles - the one on the side of the base opposite to the source triangle - to the hot end A of the target line AB.

We need not move the whole triangle ABC, as this line, the moved hot arm, is the only one needed here. So, RETURN to I.31 at line 8. Cleanup.

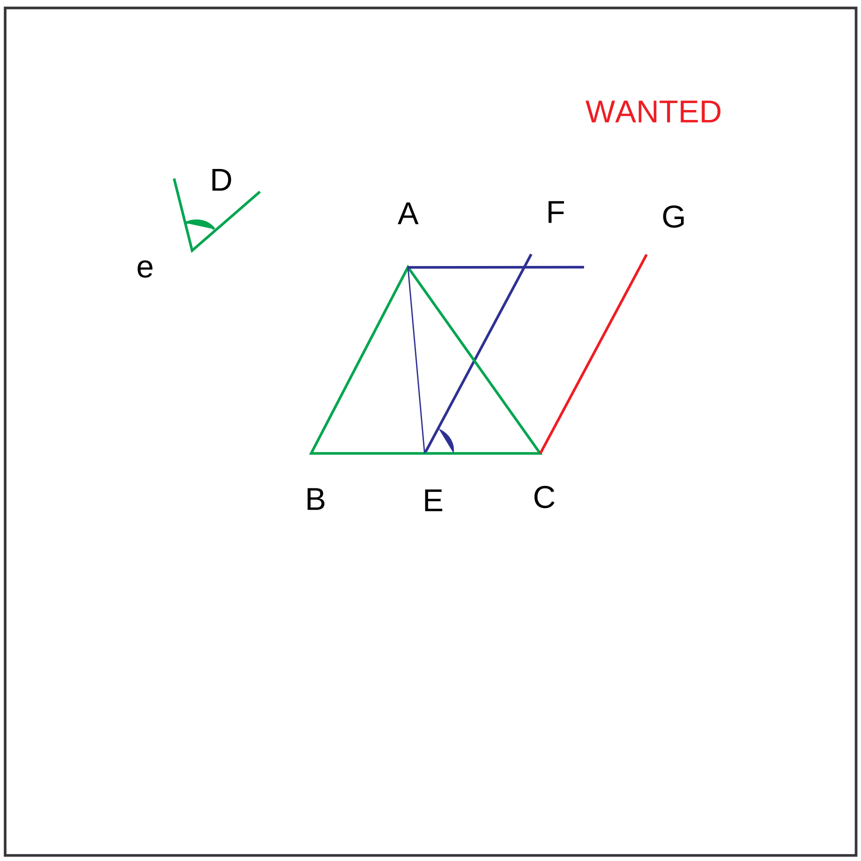


I.1:11. and let the straight line AG be produced in a straight line with eA.



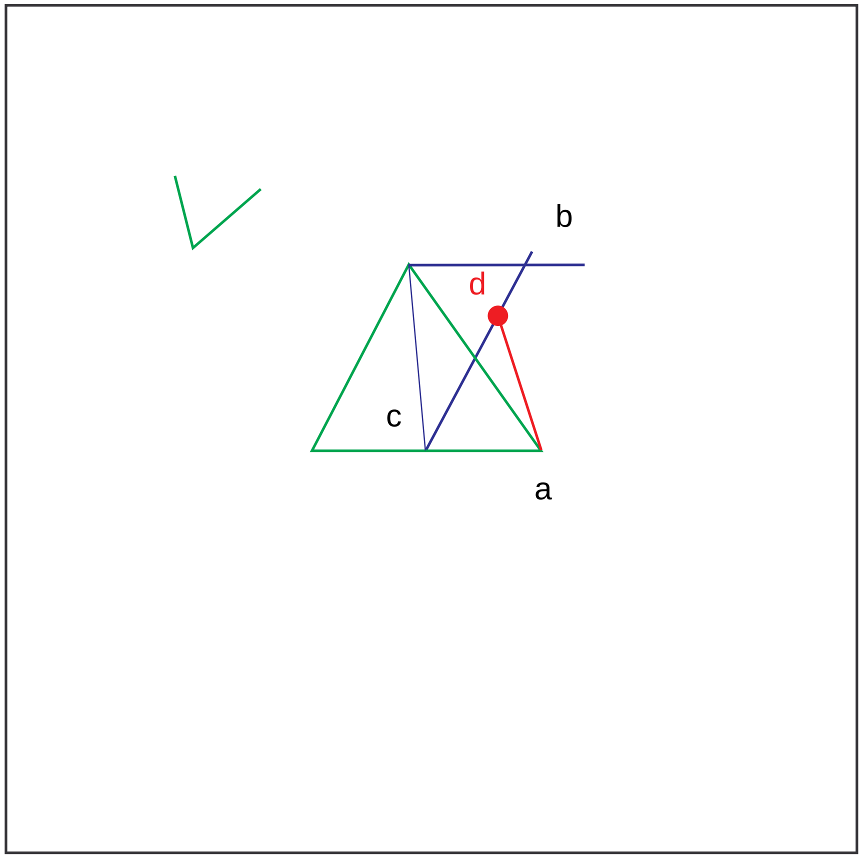
Cleanup and RETURN to I.42 at line 14.

I.42:14. and through C let CG be drawn parallel to EF.

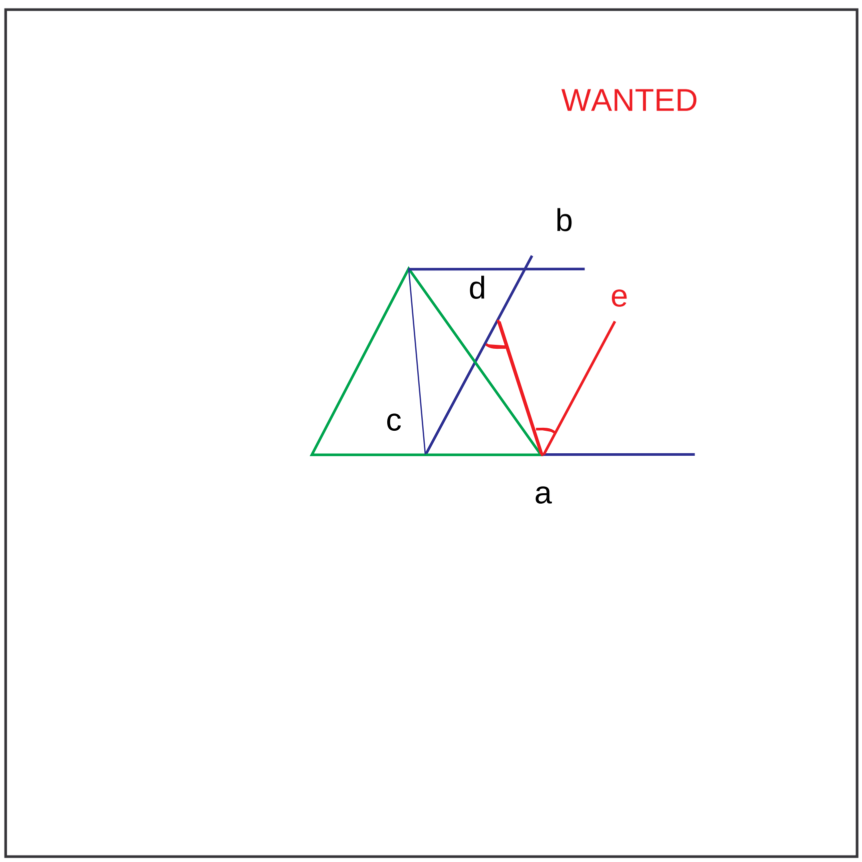


GOSUB I.31
Relabel.

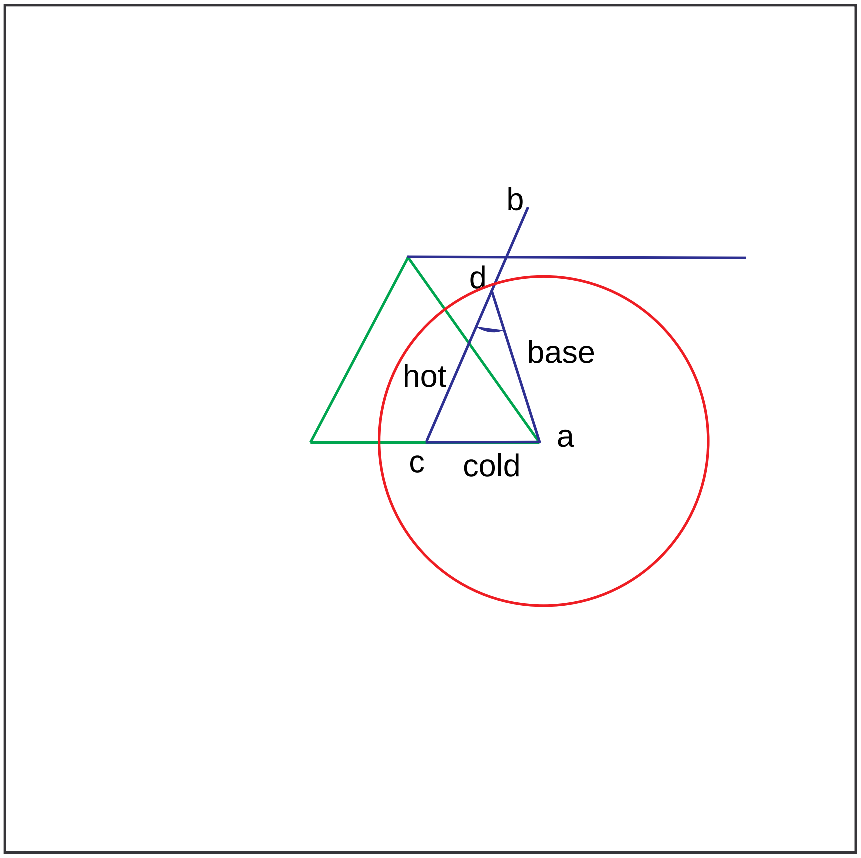
I.31:7. Let a point d be taken at random on bc, and let ad be joined.



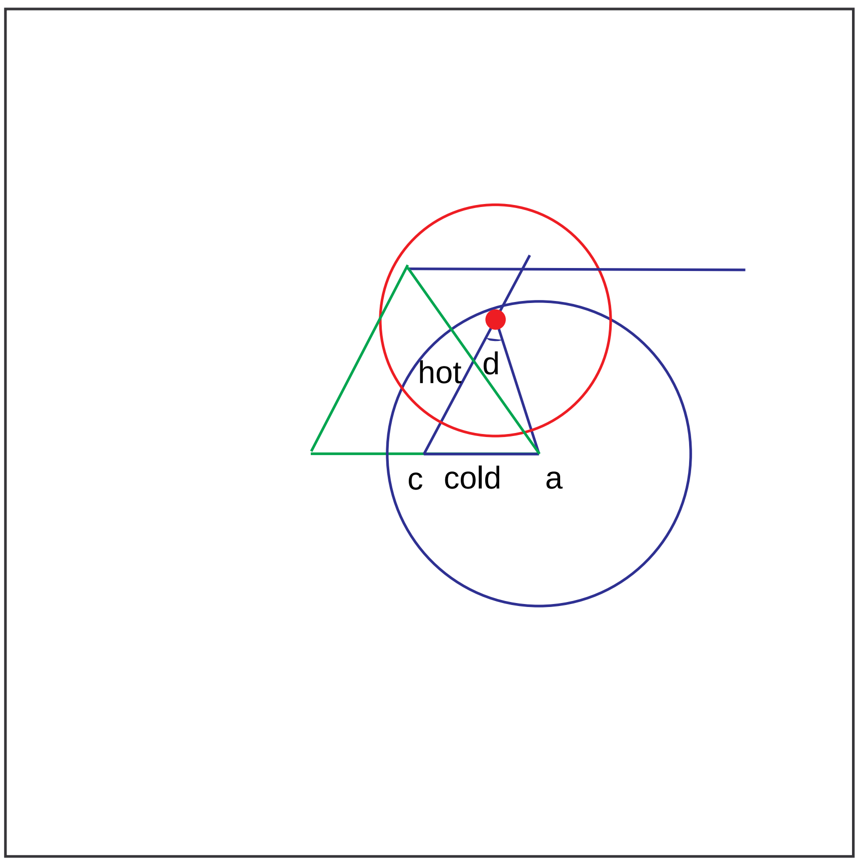
I.31:8. On the straight line da, and at the point a on it, let the angle dae be constructed equal to the angle adc [I.23];



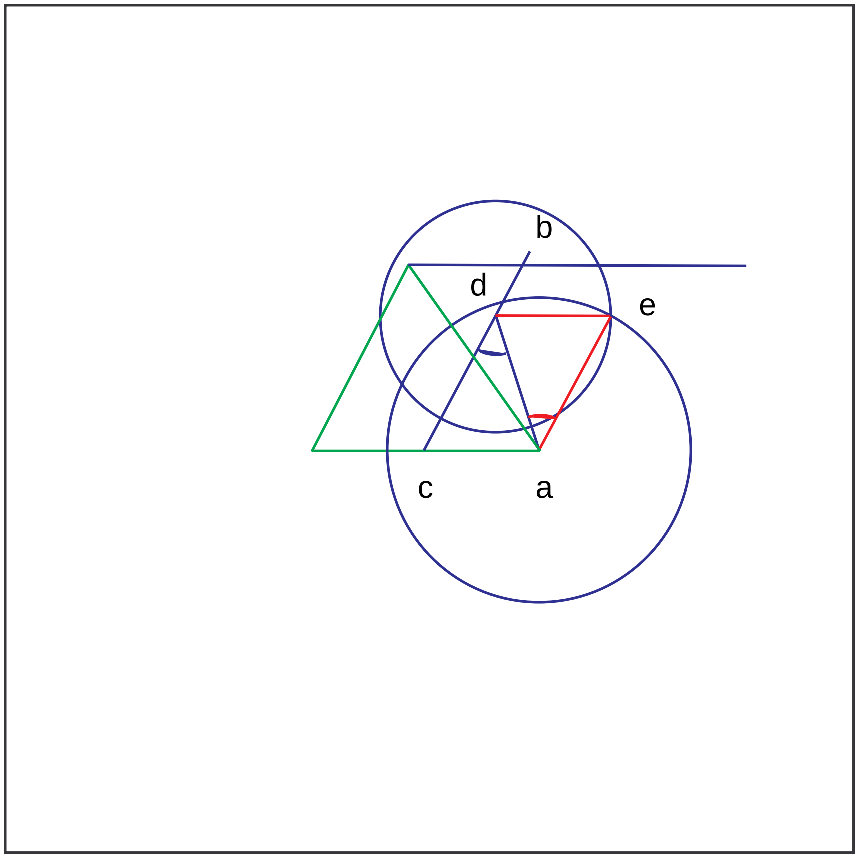
GOSUB I.23
As we have already the line ca, we GOSUB I.22P at once, to move the triangle acd. Swing the hot arm around the hot end a of the target line ad.



Swing the cold arm around the cold end d of the target line ad.



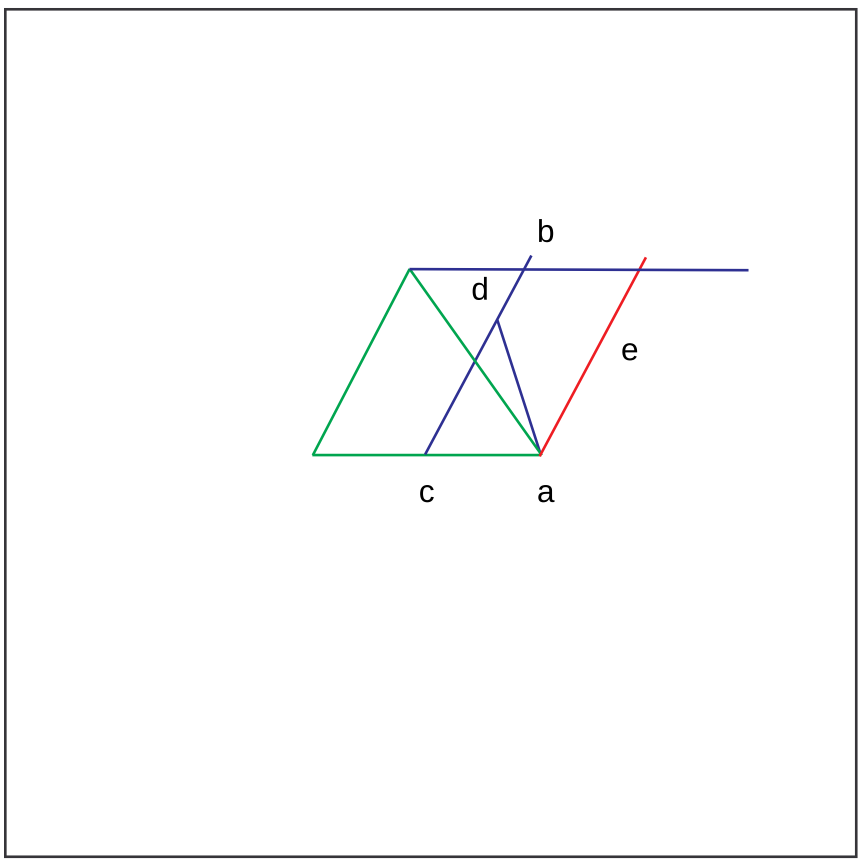
Connect the meeting point e, on the side of the target line ad opposite to the source triangle acd, to the end points of the target line. The triangle has been moved.



RETURN from I.22P to I.23.

We have the angle dae equal to the angle adc. Relabel and RETURN to I.31 at line 8.

Now we extend the new line ae which is parallel to bc, relabel, and RETURN to I.42 at line 14.



I.42:29. Therefore the parallelogram FECG has been constructed equal to the given triangle ABC, in the angle CEF which is equal to D.

Q.E.F.