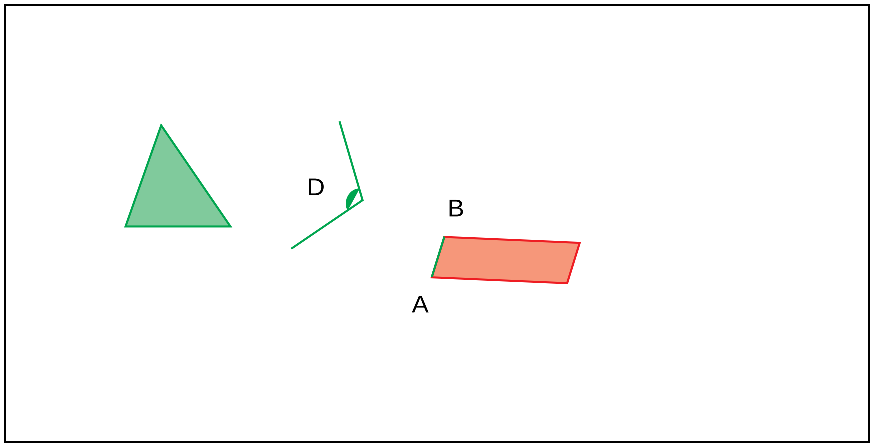
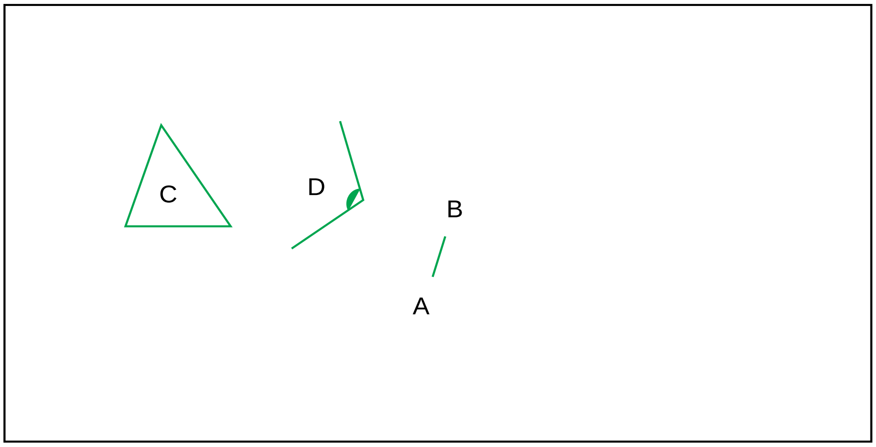
Construction 12: Book I, Proposition 44

(The Application of Areas)

To a given straight line to apply, in a given rectilineal angle, a parallelogram equal to a given triangle.

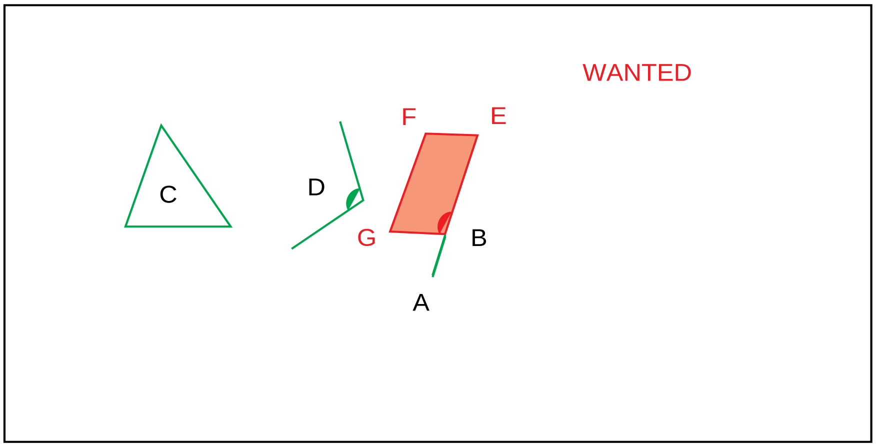


I.44:3 . Let AB be the given straight line, C the given triangle and D the given rectilineal angle.



I.44:8. Let the parallelogram BEFG be constructed equal to the triangle C, in the angle EBG which is equal to D [I.42];

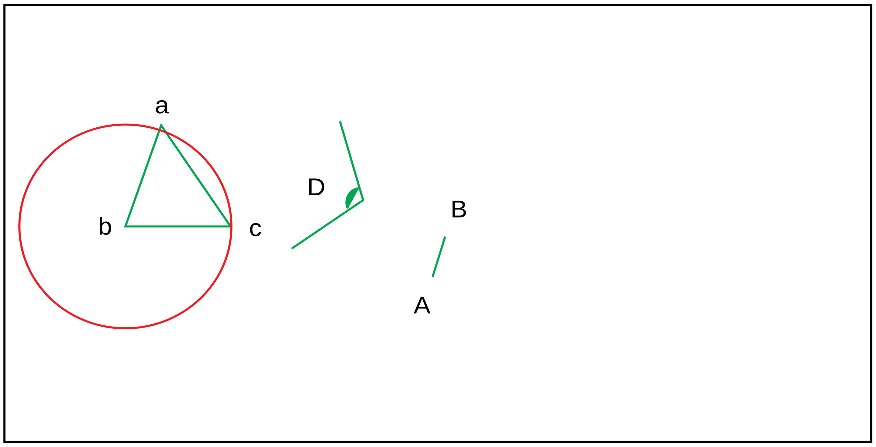
GOSUB I.42, relabel.



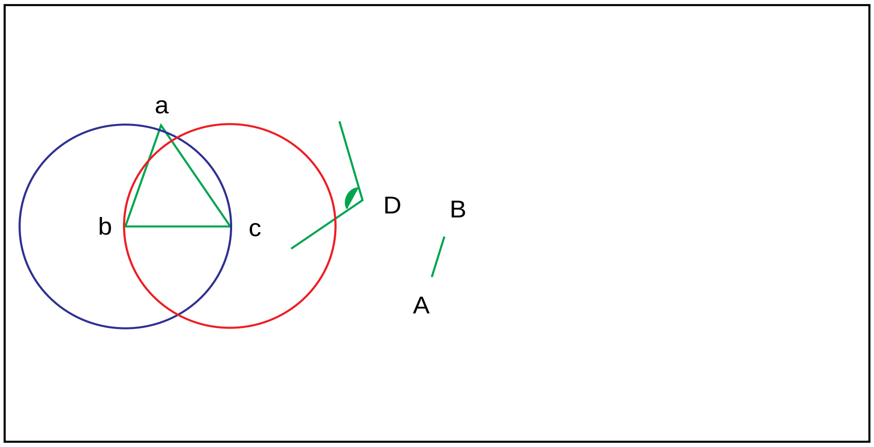
I.42:8. Let bc be bisected at e,
([I.10])

GOSUB I.10. We paraphrase I.10 to avoid relabelling.

I.10:4. Circle at b with distance bc.

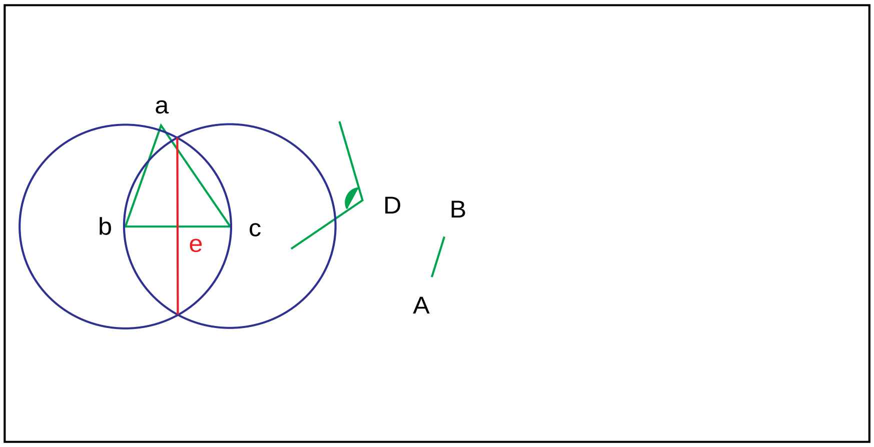


I.1:10. Circle at c with distance bc.

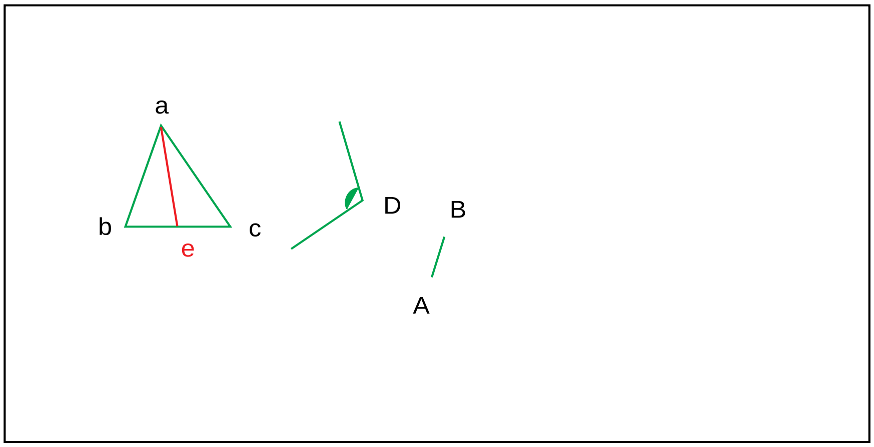


I.9:8. Connect the crossings, finding the midpoint e of bc.

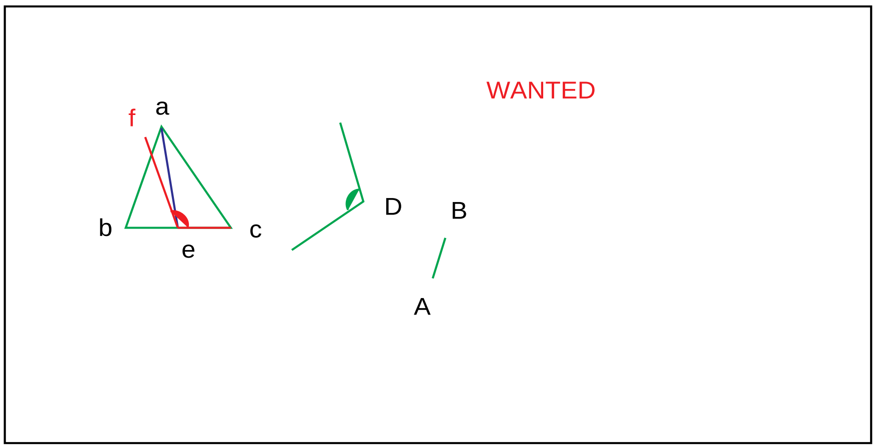
Cleanup.
RETURN to I.42 at line 8.



I.42:9. and let ae be joined; (this is not really necessary)

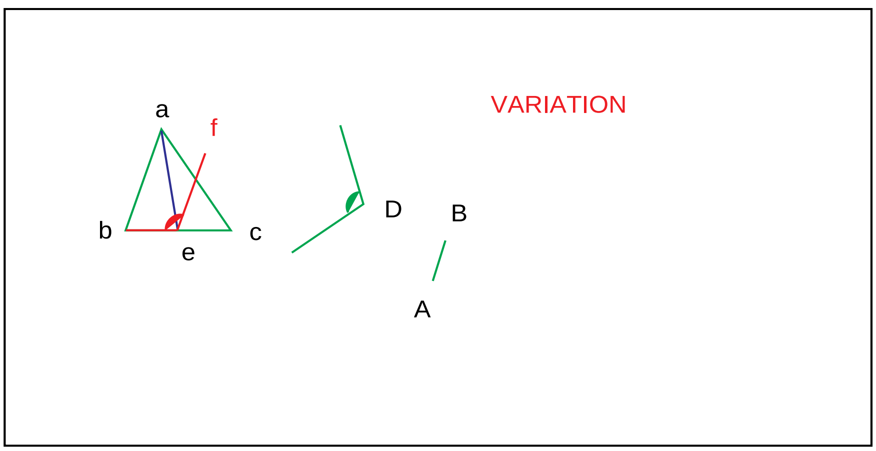


I.42:10. on the straight line ec and at the point e on it, let the angle cef be constructed equal to the angle D; [I.23]



If we follow I.42 slavishly at this point our figures will deviate significantly from Euclid's. A slight variation in I.42 is needed: move the angle D not to cef but to bef.

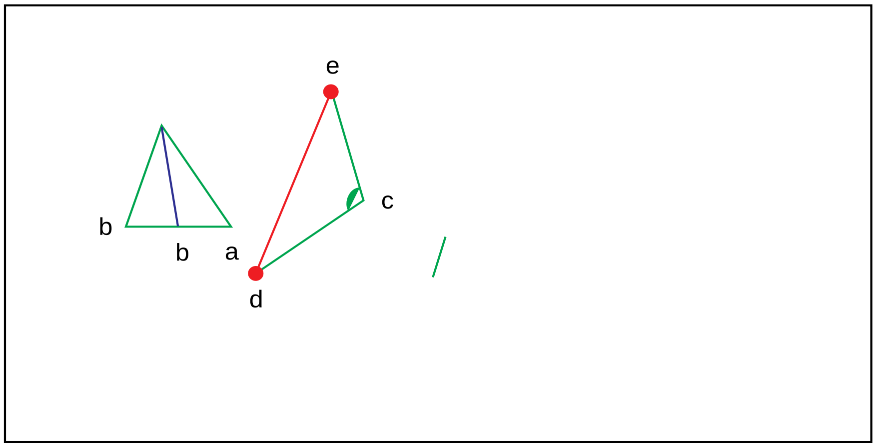
Now GOSUB I.23, relabel.



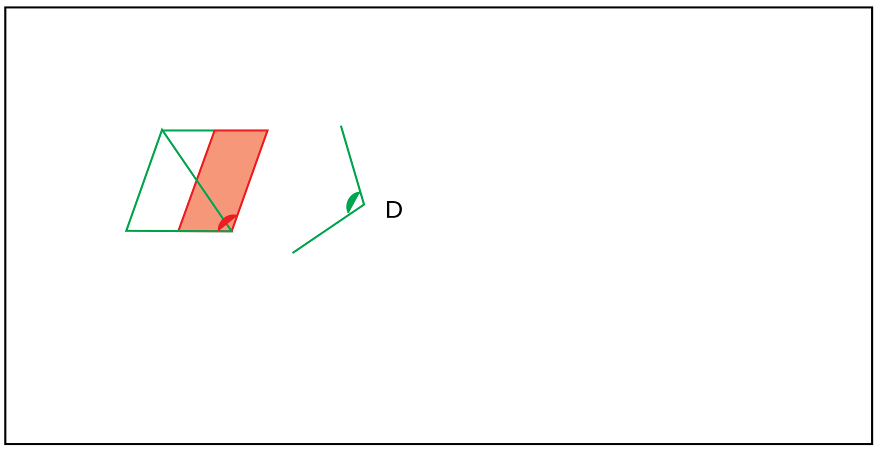
I.23:8. On the straight lines cd, ce respectively let the points d, e be taken at random;

I.23:10. let de be joined,

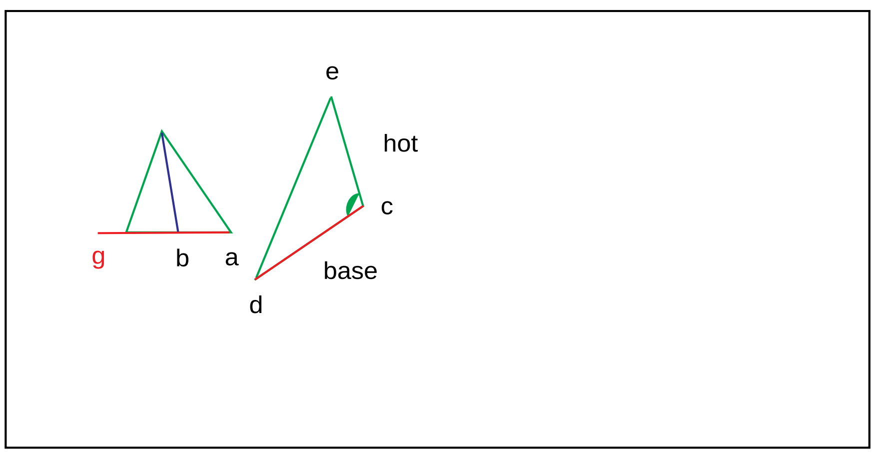
Note that a variation of the figure of I.23 is needed here: b to the left of a. Now we GOSUB I.22P to move the triangle cde so that the base dc moves onto the target line ba, the hot vertex c onto the hot end a. Compare I.23:11. Euclid is flexible!



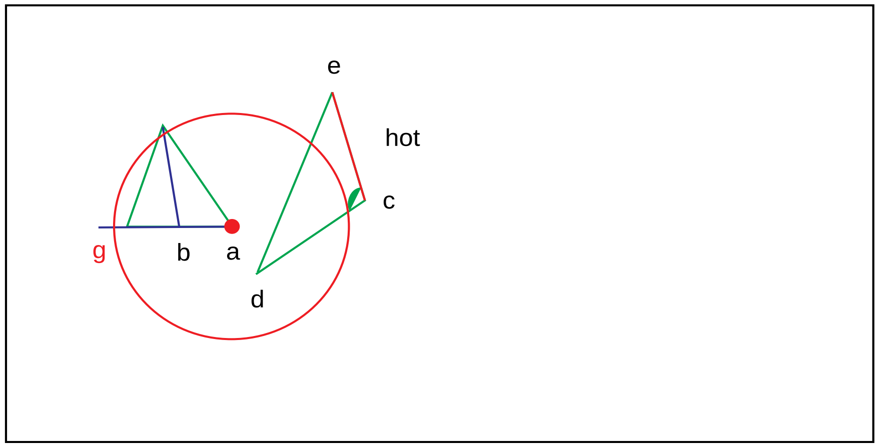
This is because we want to build, in GOSUB I.42, a rectangle equal to the triangle, and in an angle equal to D like this.



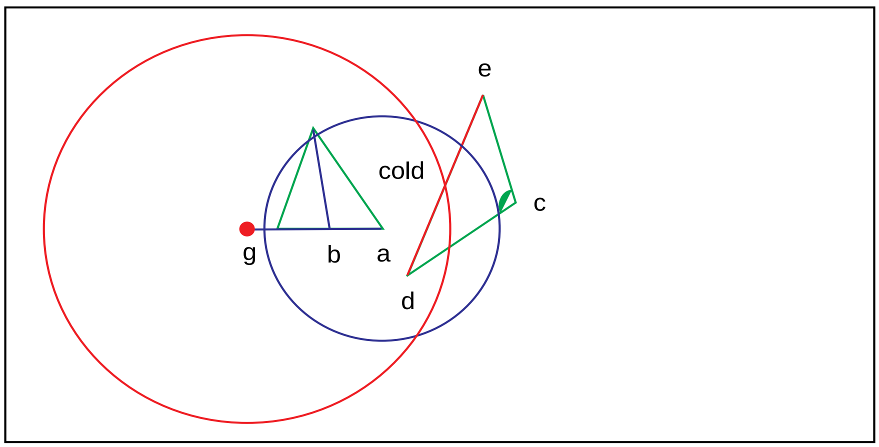
I.22P. Extend the target line and move the base, hot end, c, to hot end, a. The moved base is ag.



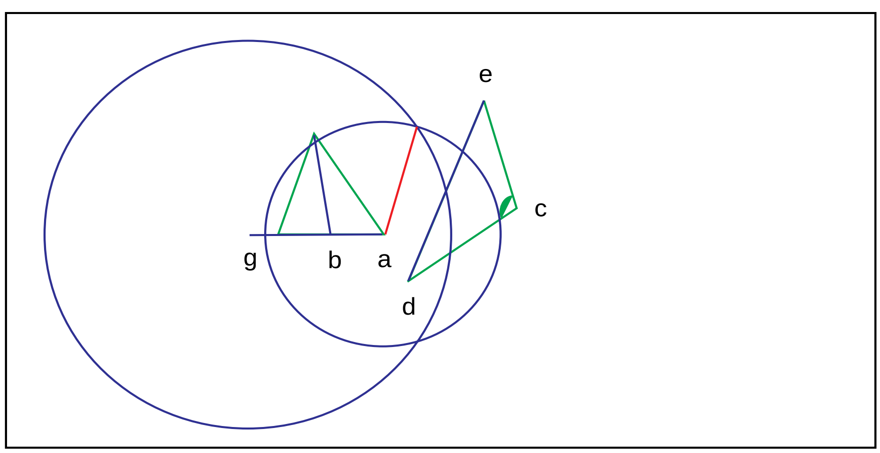
I.22P. Swing the hot arm, the distance ce, from the hot end of the moved base, a.



I.22P. Swing the cold arm, the distance de, from the cold end of the moved base, g.



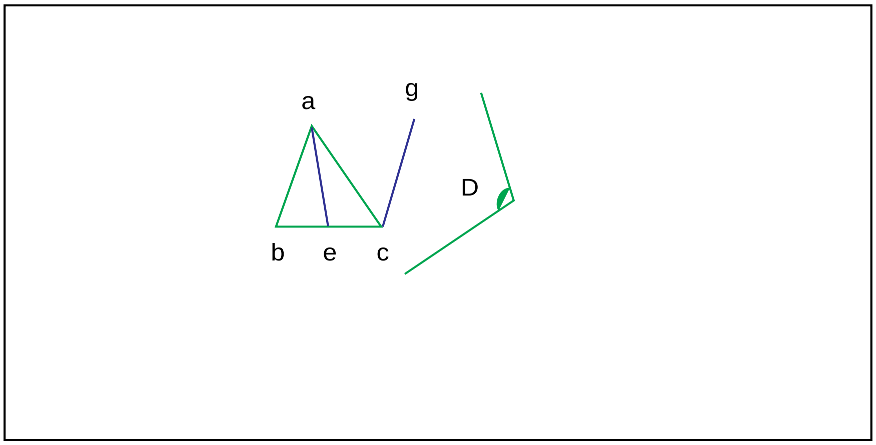
I.22P. Connect the upper intersection of the circles, h, to the moved vertex, a.



Cleanup, relabel.

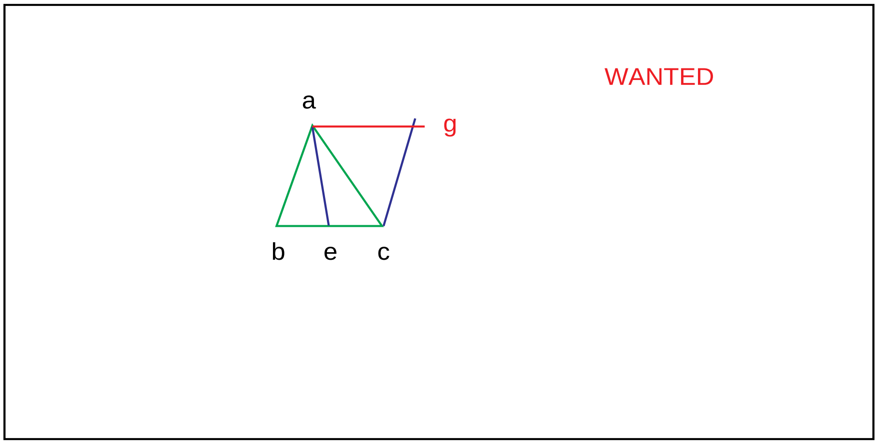
RETURN to I.42 at line 10.

Extend the moved hot side, ch, if necessary, above a.

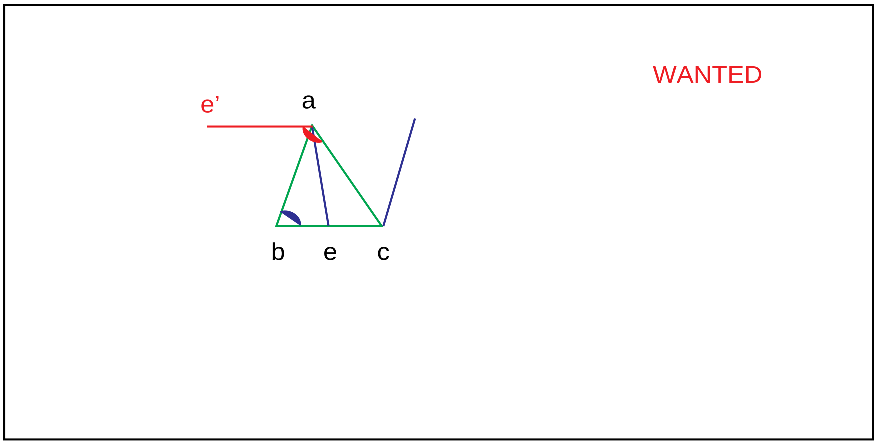


I.42:14. Through a let ag be drawn parallel to ec, [I.31]

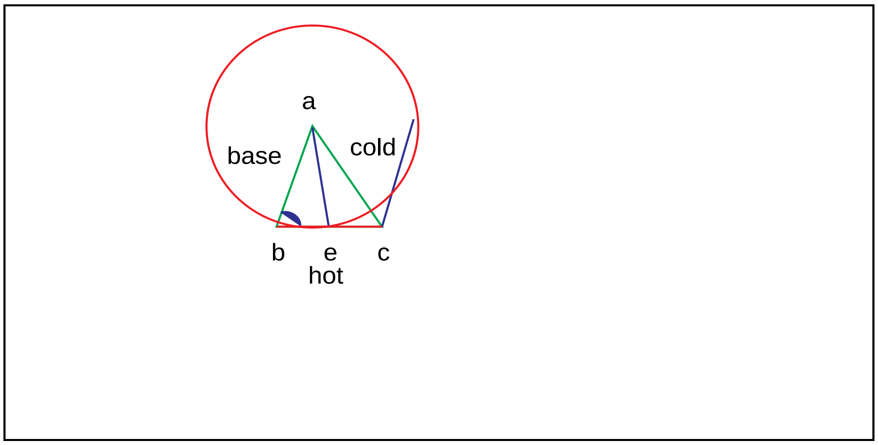
GOSUB I.31



I.31:7. Let a point d be taken at random on ec, and let ad be joined; (we will take d = b.) on the straight line ba and at the point a on it, let the angle bae' be constructed equal to the angle abc [I.23];

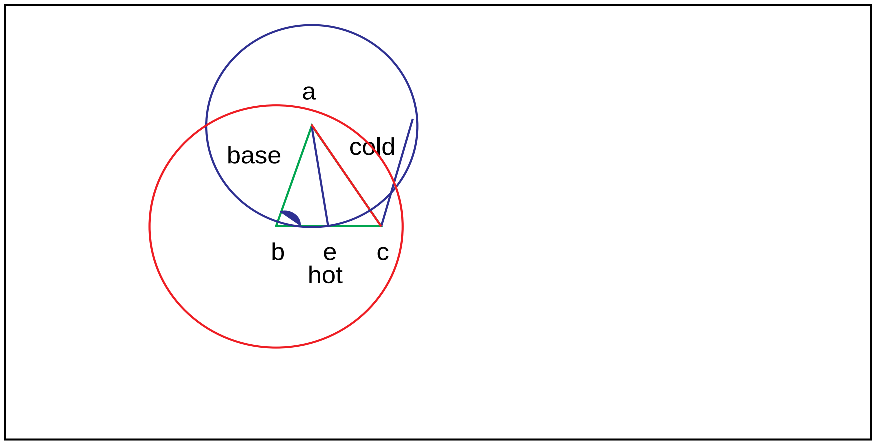


GOSUB I.23. GOSUB I.22P
We move the triangle abc. The base ab is not moved, but the hot end b is moved to the other end, a.

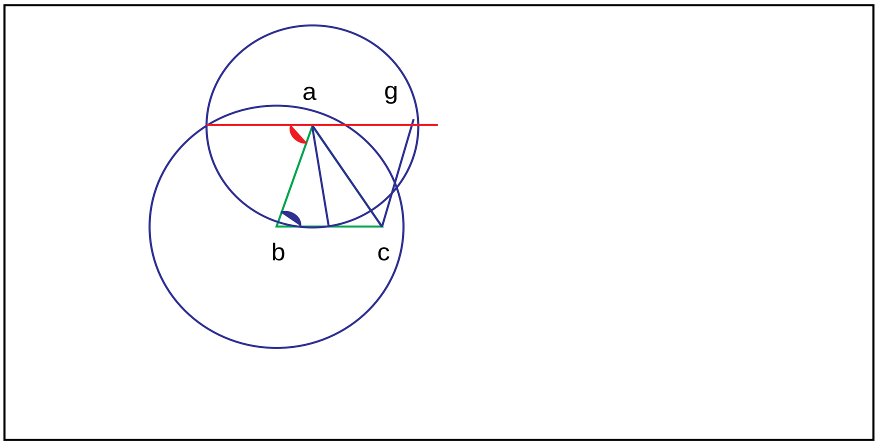


Rotate the hot arm around a.

Rotate the cold arm around b.



Connect the meeting point to a. Extend this line to the right to meet cg at g.

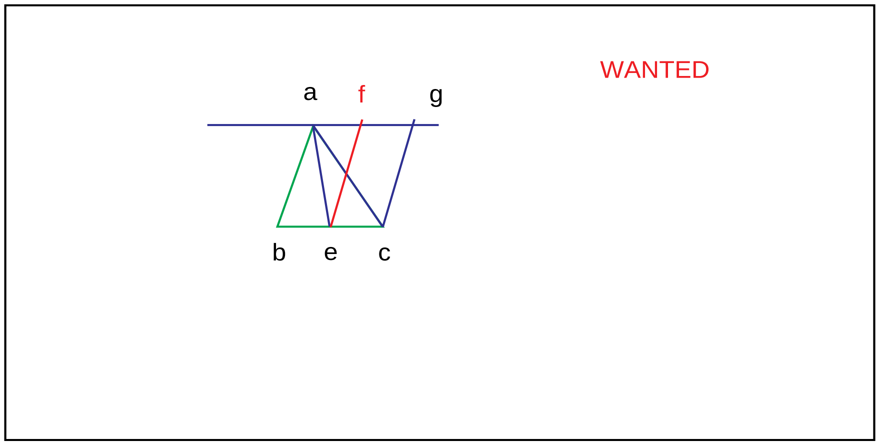


RETURN to I.31 at line 7 and cleanup.

RETURN to I.42 at line 14.

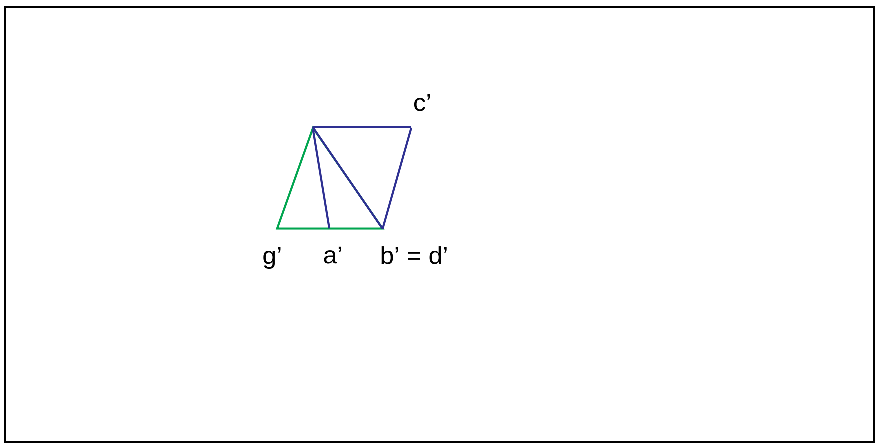
I.42:14. and through e let ef be drawn parallel to cg.

We have relabeled within Euclid's text here, as we must now draw the fourth side of the parallelogram - a slight variation of I.42 is needed for I.44.



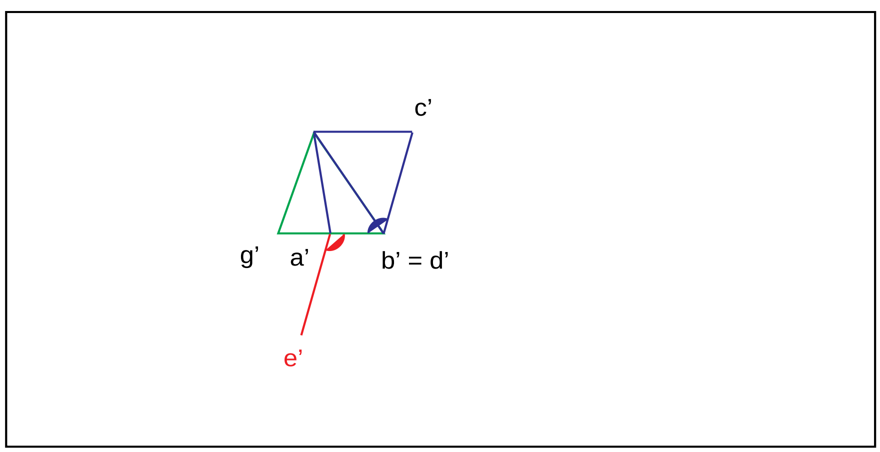
GOSUB I.31. Relabel.

I.31:7. Let a point d' be taken at random on b'g', and let a'd'be joined. We will choose d' = b'.

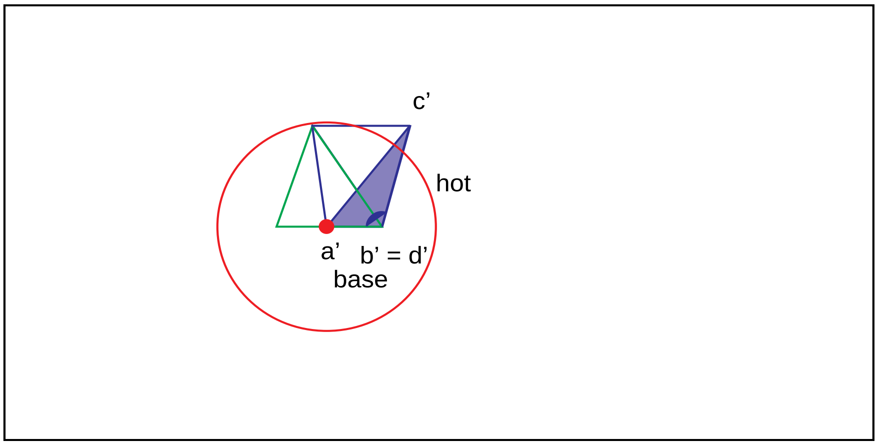


I.81:8. on the straight line d'a', and at the point a' on it, let the angle d'a'e' be constructed equal to the angle a'b'c' [I.23];

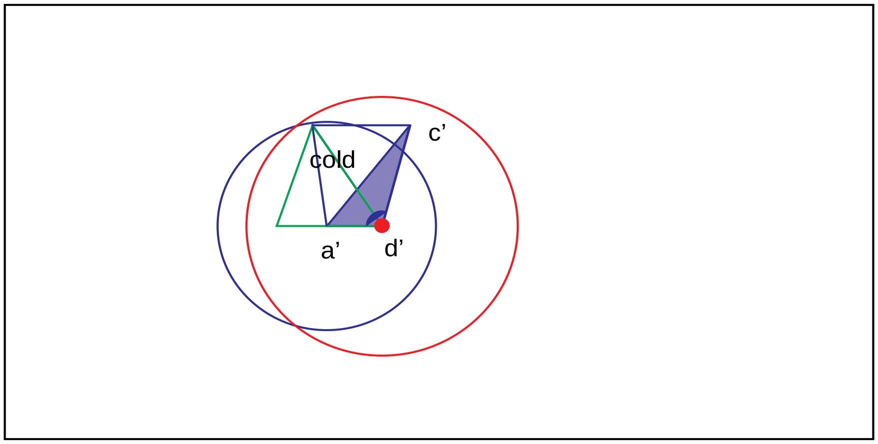
GOSUB I.23. GOSUB I.22P. Relabel (base, hot, cold). Note again the special case: the base does not move.



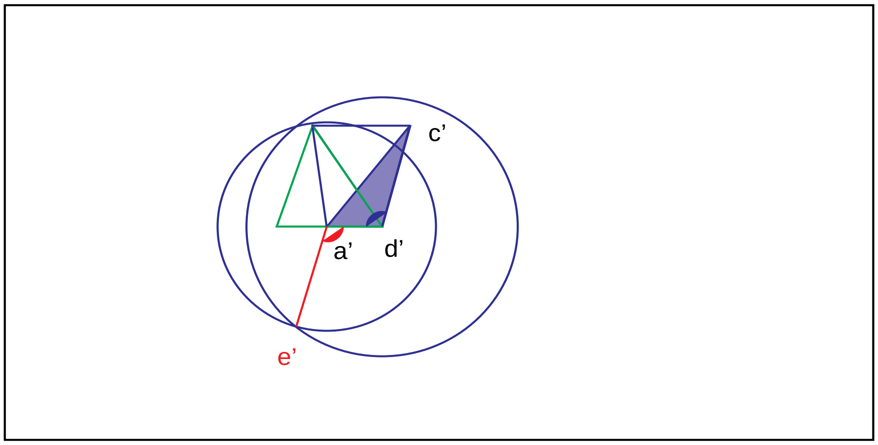
Move the hot arm, swing it around the point a’.



Move the cold arm, swing it around the point d’.



Let e’ be the point where the two circles meet, on the opposite side of the base. Connect e’a’.

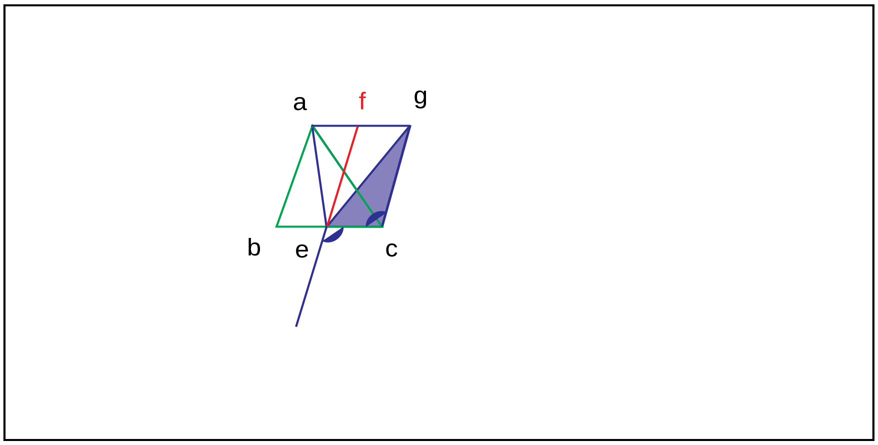


Now we have the angle d'a'e' equal to the angle a'd'g'.

RETURN to I.23 at line 8. Relabel.

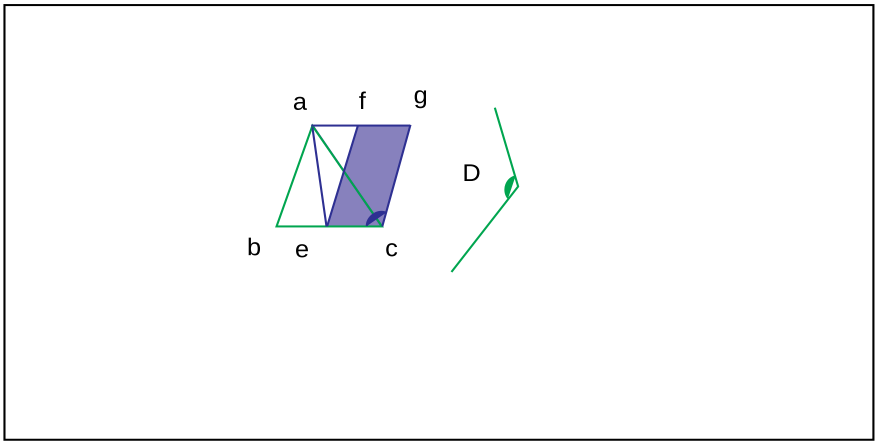
The new line is parallel to cg. Extend it upwards to meet the line ag at f.

RETURN to I.42 at line 14. Cleanup.



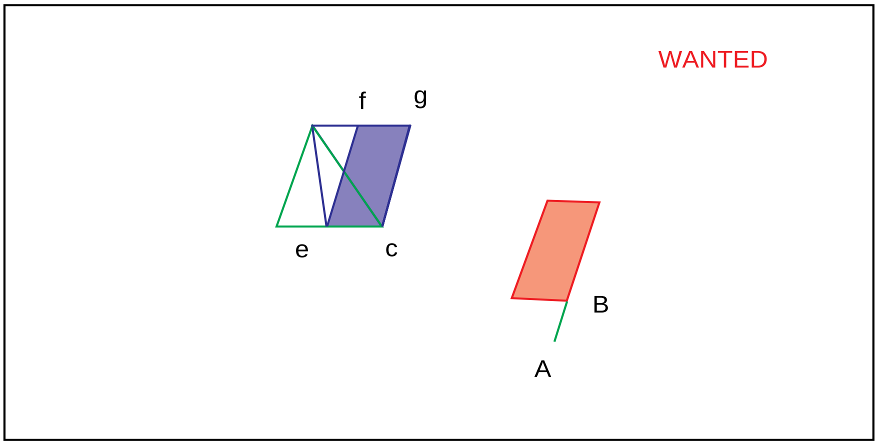
I.42:14. The parallelogram fecg has been constructed equal to the given triangle abc, in the angle ecg which is equal to D.

RETURN to I.44 at line 9.

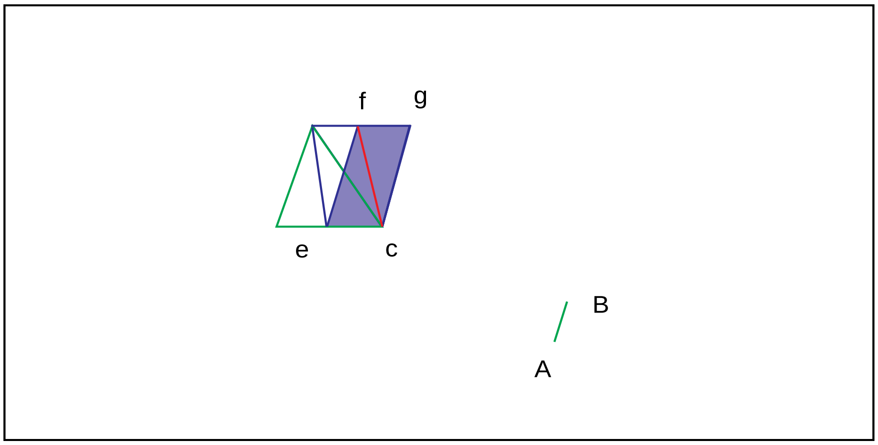


I.44:10. let it be placed so that eg is in a straight line with AB;

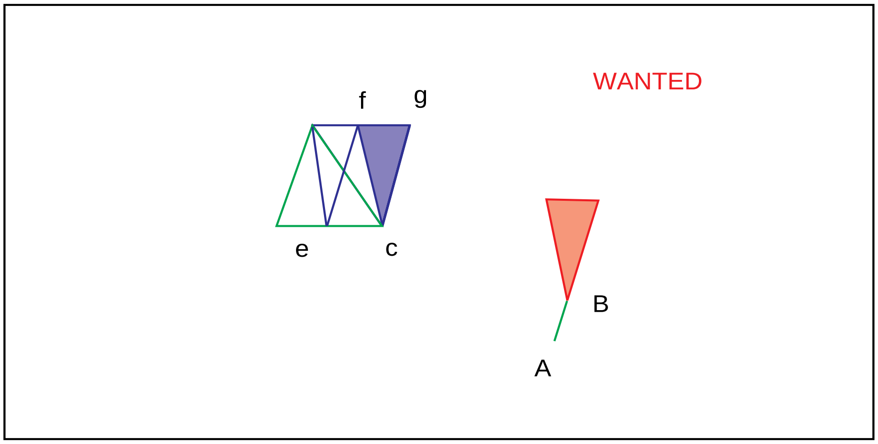
Here we have a new problem: to move a parallelogram! Looking ahead to C#13 < I.45, the next construction, we see a way: divide the parallelogram into two triangles. Move each triangle with I.22. This construction could have followed I.22.



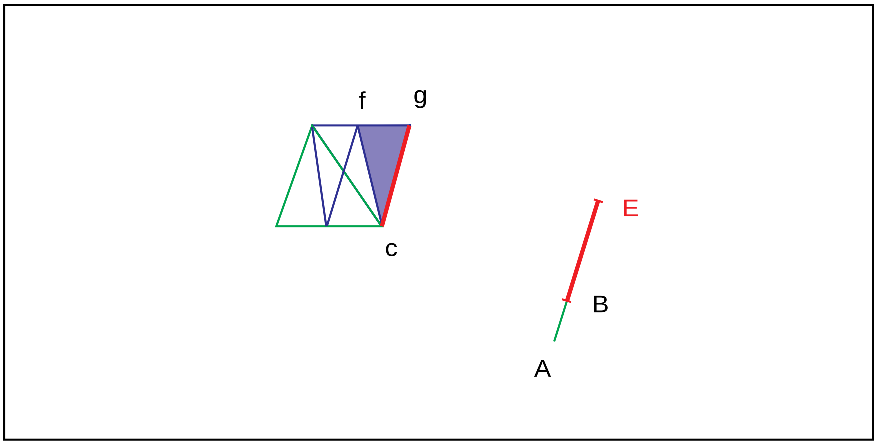
Divide the rectangle with the line cf.



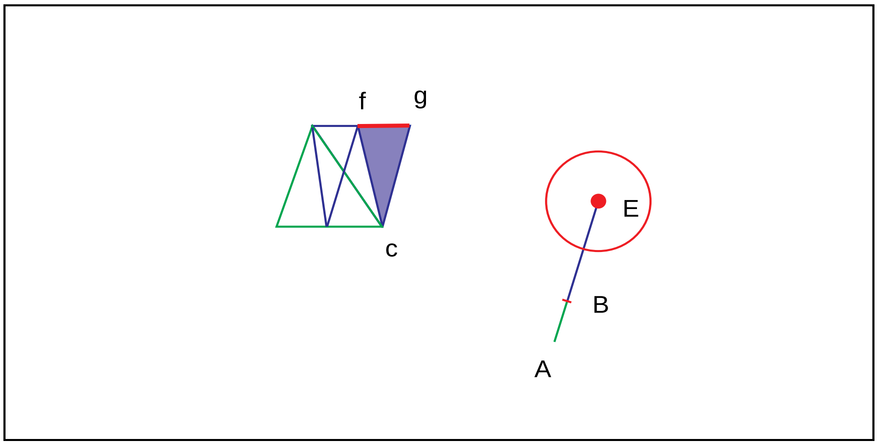
To move the triangle cfg so that the line cg moves to the line BE which extends AB.



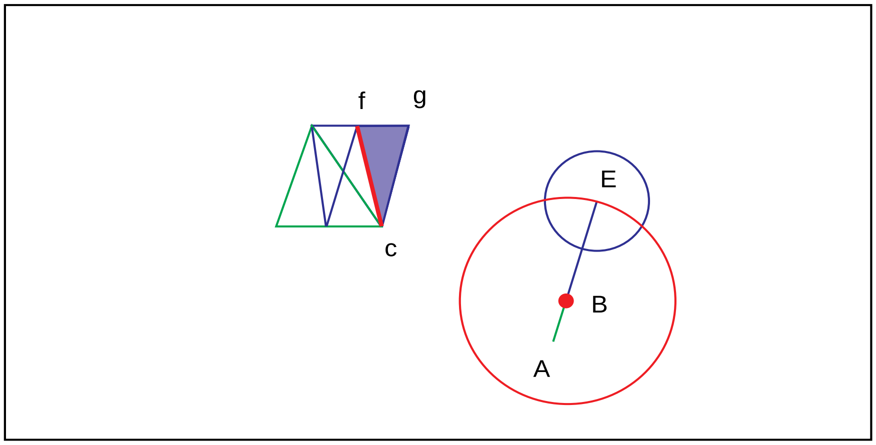
Extend AB upwards and move cg onto BE. ([I.3, dividers.])



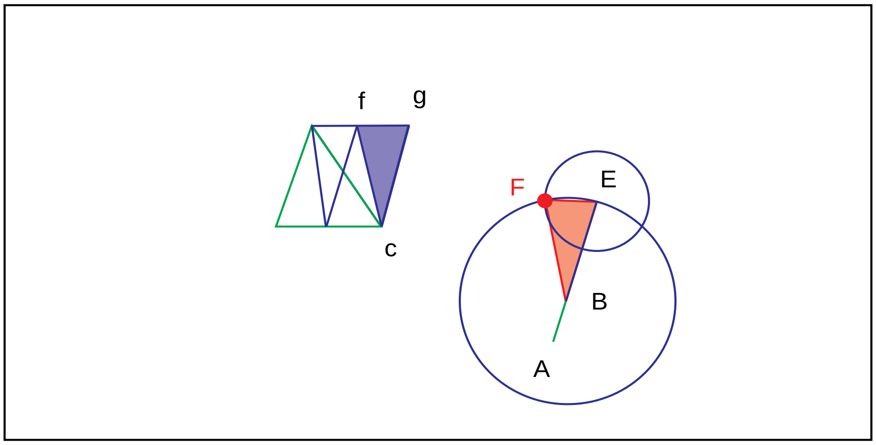
Move fg, swing it around E.



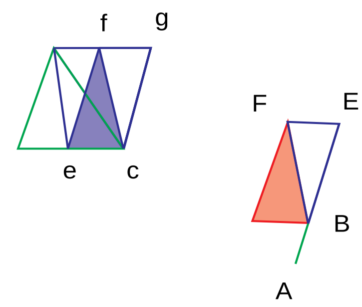
Move cf, swing it around B.



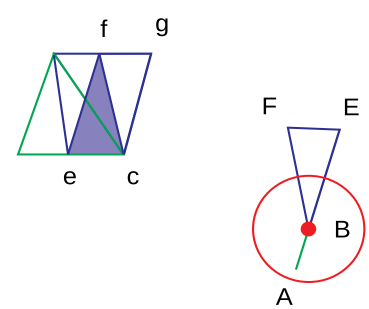
Locate the point F at the meeting point of the two circles to the left of BE. Connect FE, FB.



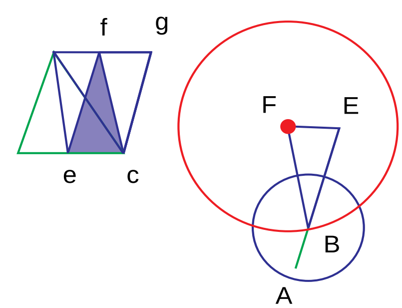
To move the triangle cef so that the line fc moves onto FB.



Move the line ec, swing it around B.

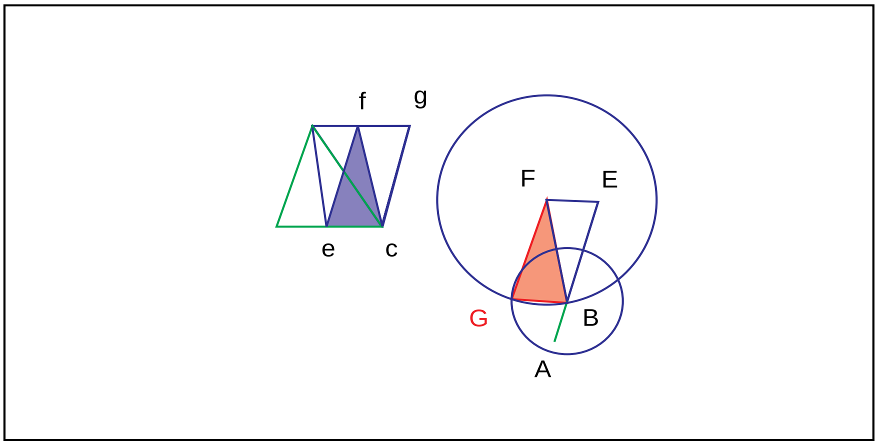


Move the line ef and swing it around F.

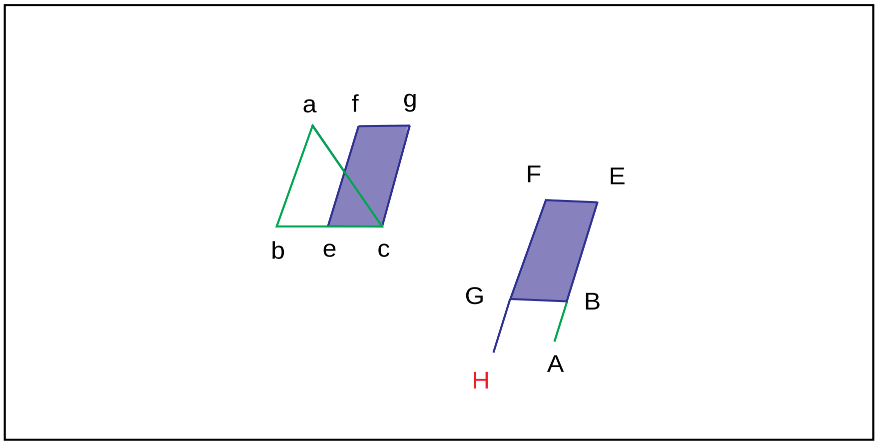


Locate the point G at the meeting point. Connect GB, GF.

RETURN to I.44 at line 10.
Cleanup.



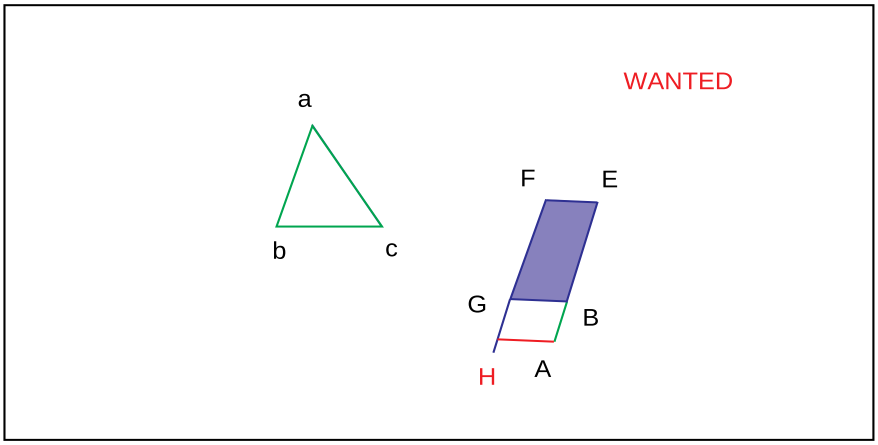
I.44:10. let FG be drawn through to H, (that is, extend FG downwards)



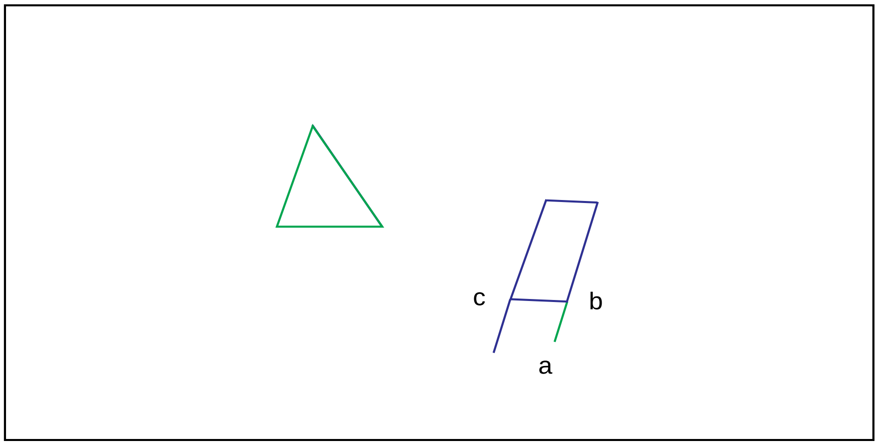
I.44:11. and let AH be drawn through A parallel to either BG or EF.[I.31]

Note that the point H has not yet been located. We are to construct the parallel line through A, then see where it meets the extension of FG to locate H. We will construct a parallel to BG; rather than EF, as BG is closer to A. GOSUB I.31.

Relabel.



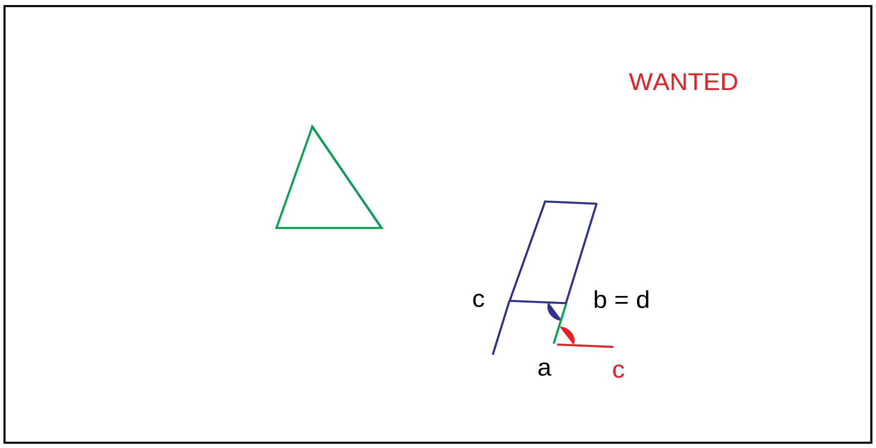
I.31:7. Let a point d be taken at random on bc, and let ad be joined. We will take d = b, so ad = ab is already joined.



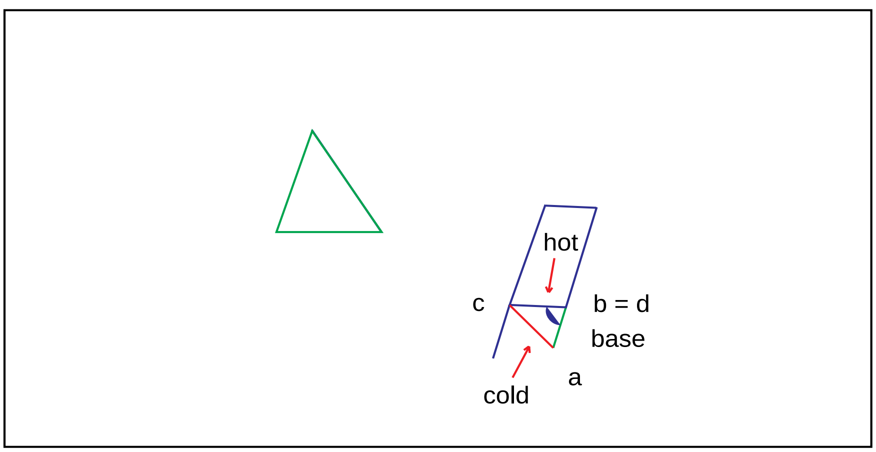
I.31:8. on the straight line da, and at the point a on it, let the angle dac be constructed equal to the angle adc [I.23];

GOSUB I.23

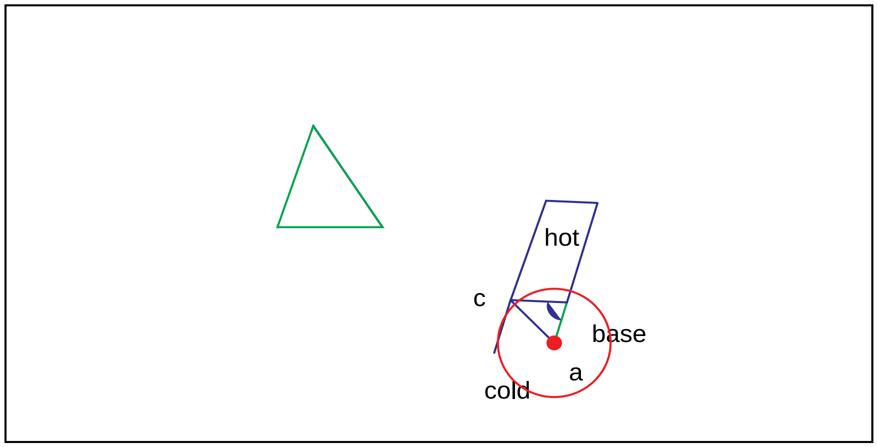
GOSUB I.22P
Relabel.



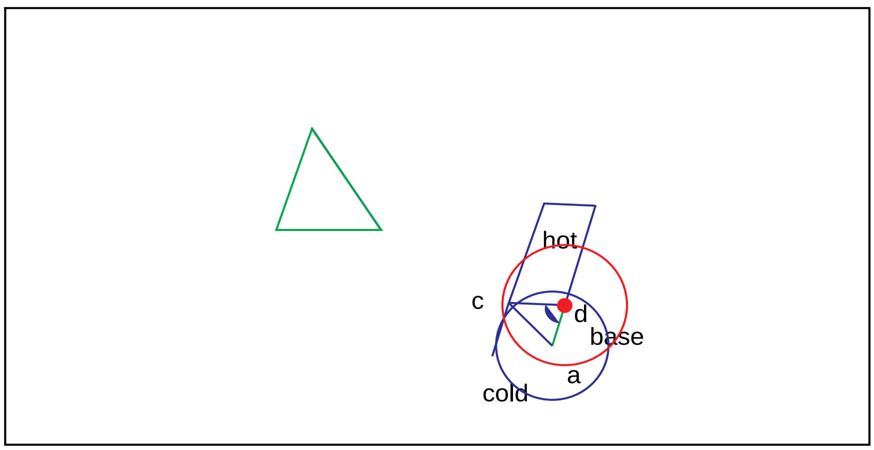
Connect ac. (See step 13 above. Again, the base does not move.)



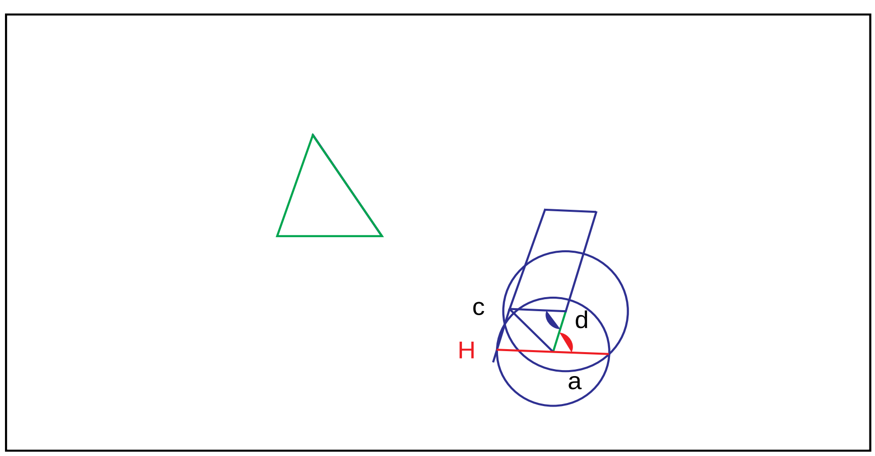
Move the hot arm, swing it.



Move the cold arm, swing it.



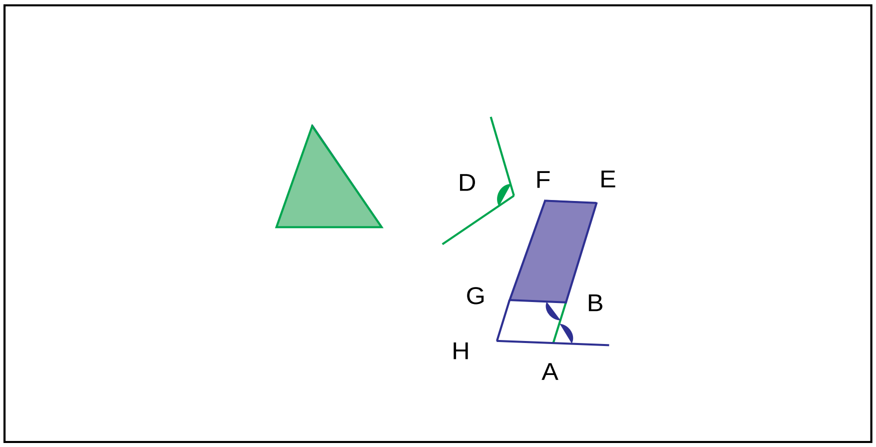
Connect the lower end of the base to the meeting point on the right. Extend it left, locating H.



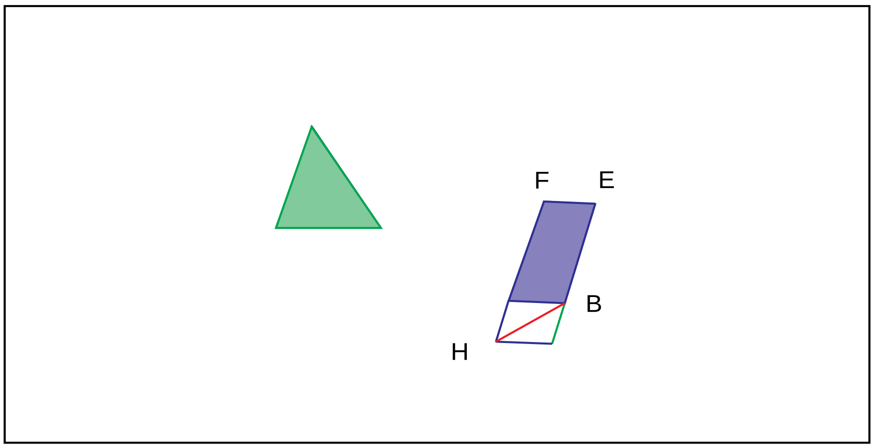
RETURN to I.23.

RETURN to I.31.

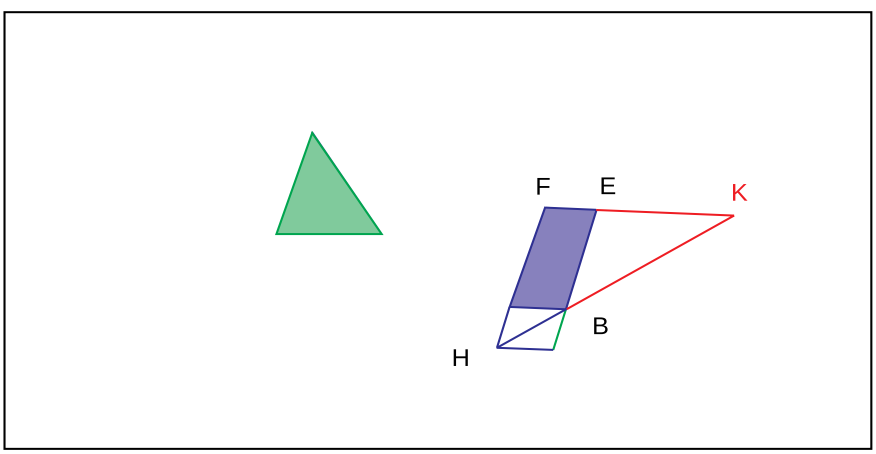
RETURN to I.44 at line 11.
Cleanup.



I.44.13. Let HB be joined.

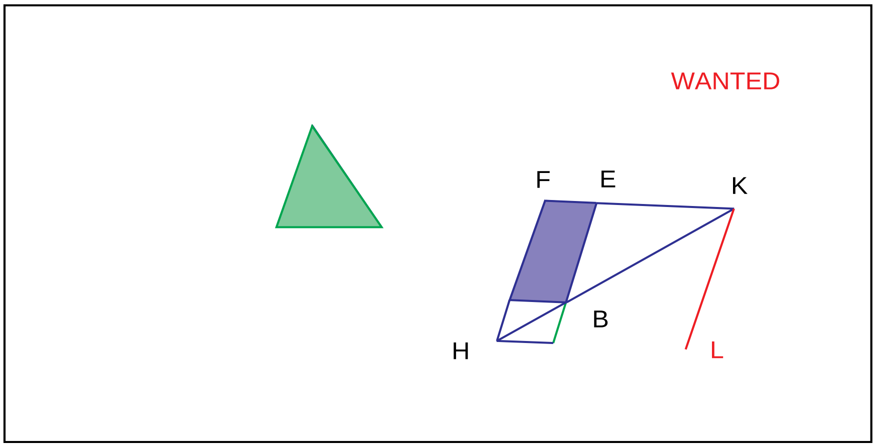


I.44:20. ...[Post. 5] -- Therefore HB, FE, when produced, will meet. Let them be produced and meet at K;

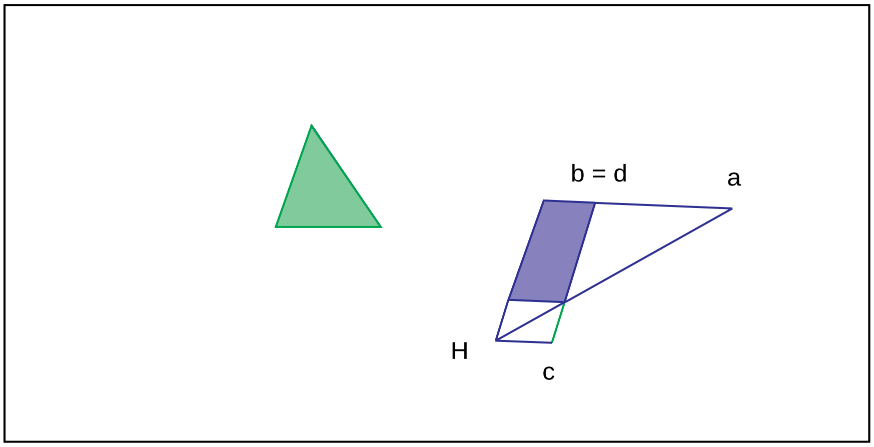


I.44:22. through the point K let KL be drawn parallel to either EA or FH, [I.31]

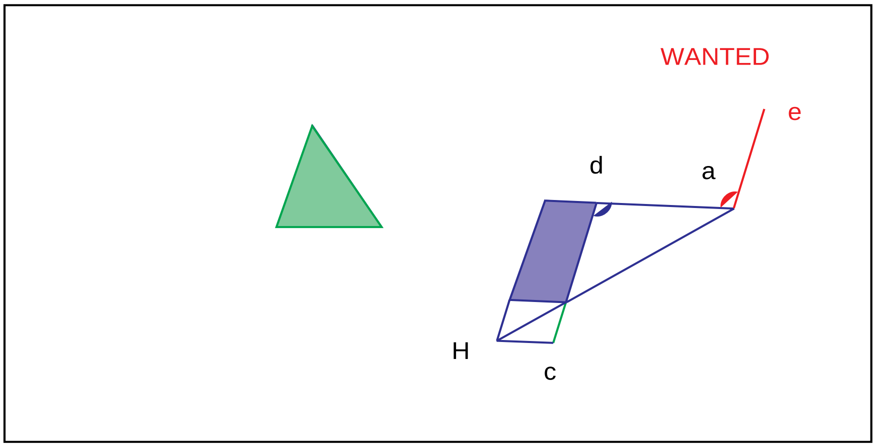
GOSUB I.31. One more (the seventh and final) time. As usual the point L is not yet determined. We will use EA as it is closer to K. Relabel.



I.37:7. Let a point d be taken at random on bc, and let ad be joined. (We choose d = b. We have ab=dc already.)

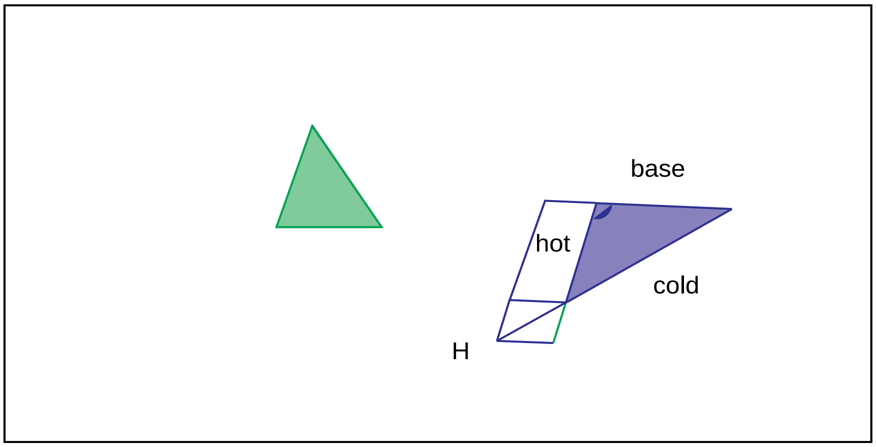


I.31:8. on the straight line da, and at the point a on it, let the angle dae be constructed equal to the angle adc [I.23];

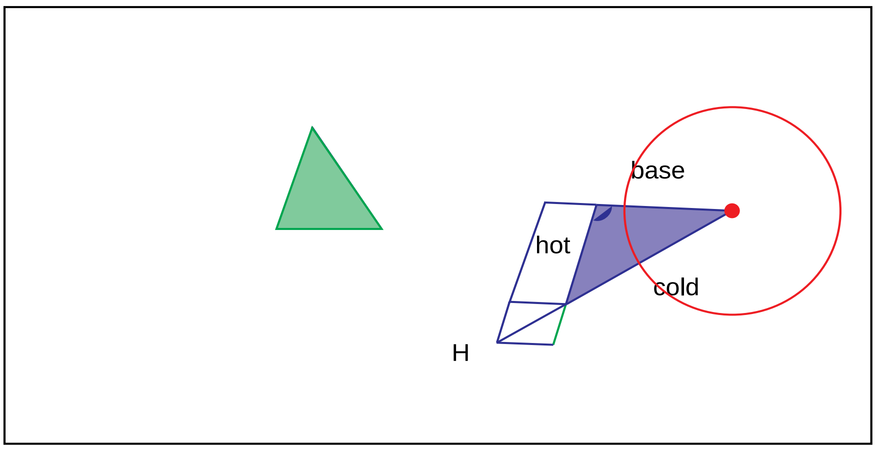


GOSUB I.23. We must choose points on the lines enclosing the angle to be moved, adc, connect them to make a triangle, then move the triangle with I.22P. We already have a triangle, so let's use it. Relabel base, hot, cold points.

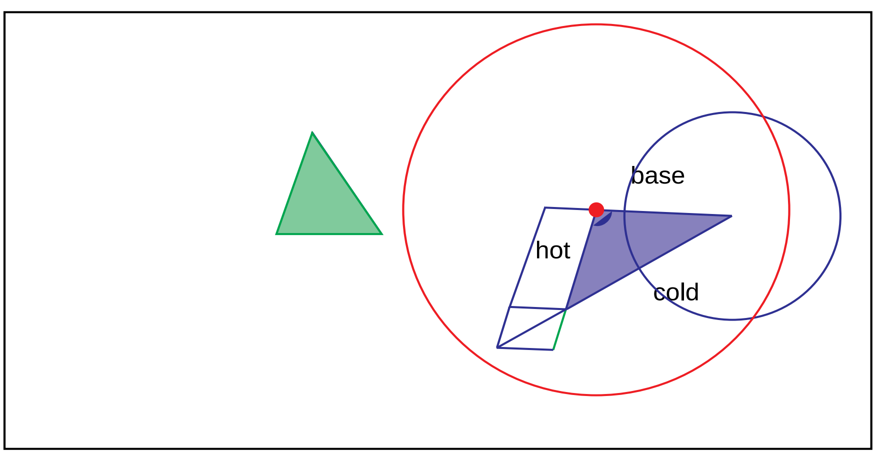
GOSUB I.22P. Again, the base does not move.



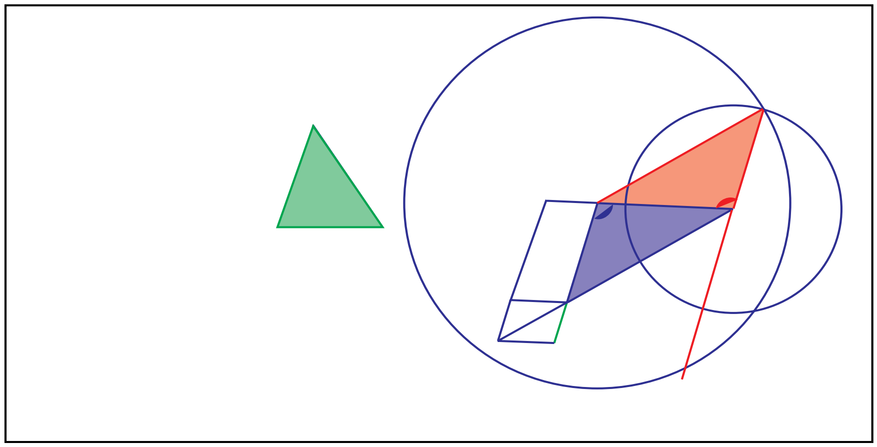
Move the hot arm to the other end of the base, swing it.



Move the cold arm to the other end of the base, swing it.



The hot and cold circles meet in two points. Choose the point on the side of the base opposite from the triangle being moved. Connect it to the centers of the circles. Extend the moved hot arm downwards.

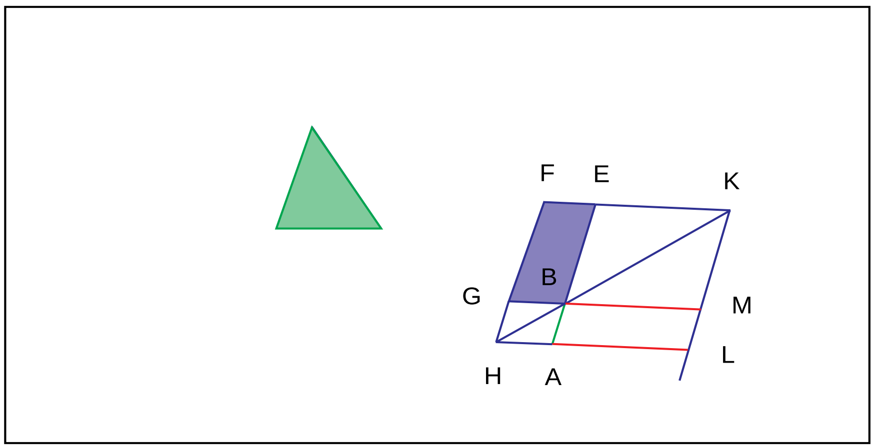


RETURN to I.23.
RETURN to I.31.
RETURN to I.44 at line 22.
Cleanup.
Relabel.

I.44:24. and let HA, GB be produced to the points L, M.

I.44:34. Therefore the parallelogram LB equal to the triangle C has been applied to the given straight line AB, in the angle ABM which is equal to D.

Q.E.F.



This is the Mount Everest of Book I, or perhaps, of the entire Euclidean (Pythagoraen, Egyptian, Atlantean ?) landscape. With 61 drawings portraying 36 constructive steps, we have bridged from ancient geometry to medieval algebra: geometric algebra! But is there a shorter route? Recall that after step 16), we came to a crux: to move the parallelogram built on half of the base of the triangle. We had been following Eucl;id's tracks slavishly. What if we moved the triangle first?