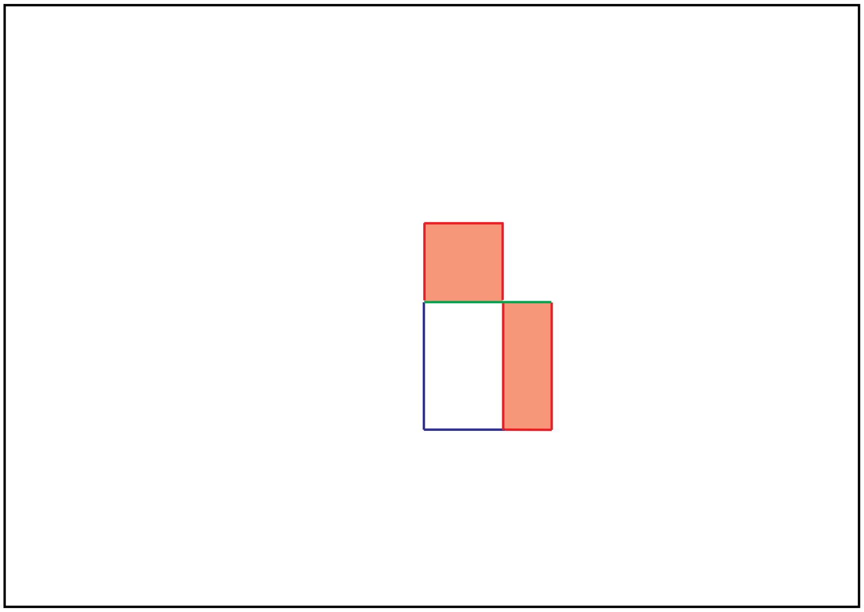
# Construction 15: Book II, Proposition 11

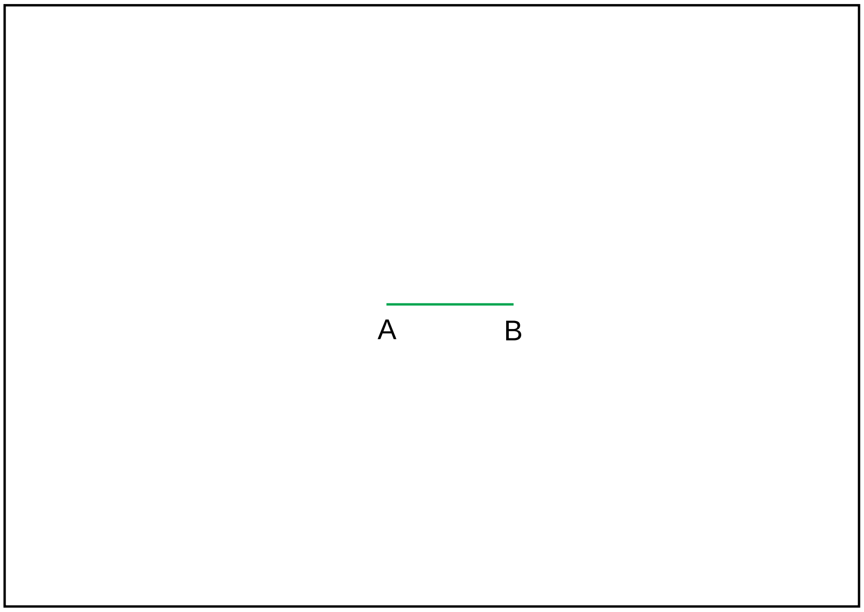
*NOTE*: This is DEMR, the golden section (see also the division in extreme and mean ratios, VI.30, needed for the pentagram).
It is a part of geometrical algebra: to solve geometrically the algebraic equation, $x^2 + ax = a^2$.

*To cut a given straight line so that the rectangle contained by the whole and one of the segments is equal to the square on the remaining segment.*

*NOTE*: We will interpret this instruction as: to cut *and* to draw the rectangle and the small square:

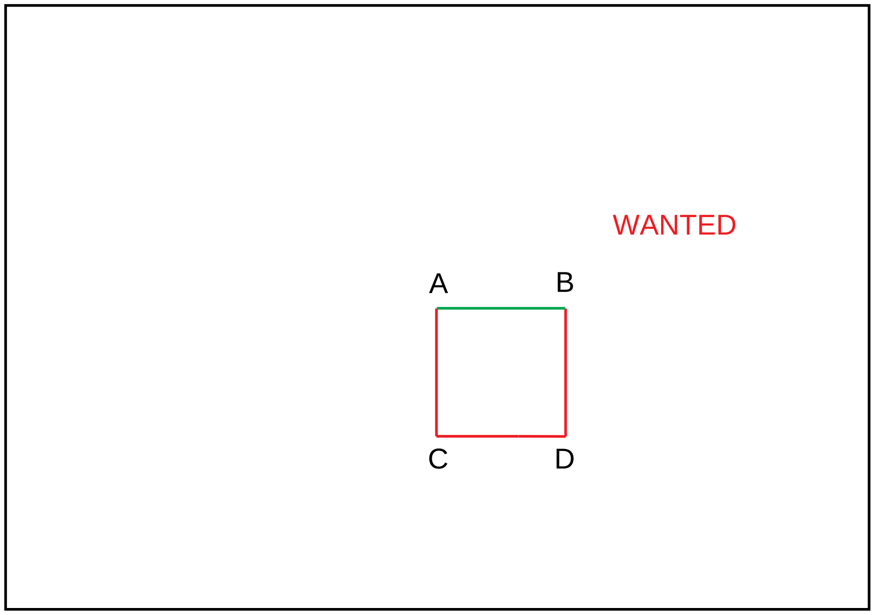


I.11:4. Let AB be the given straight line;



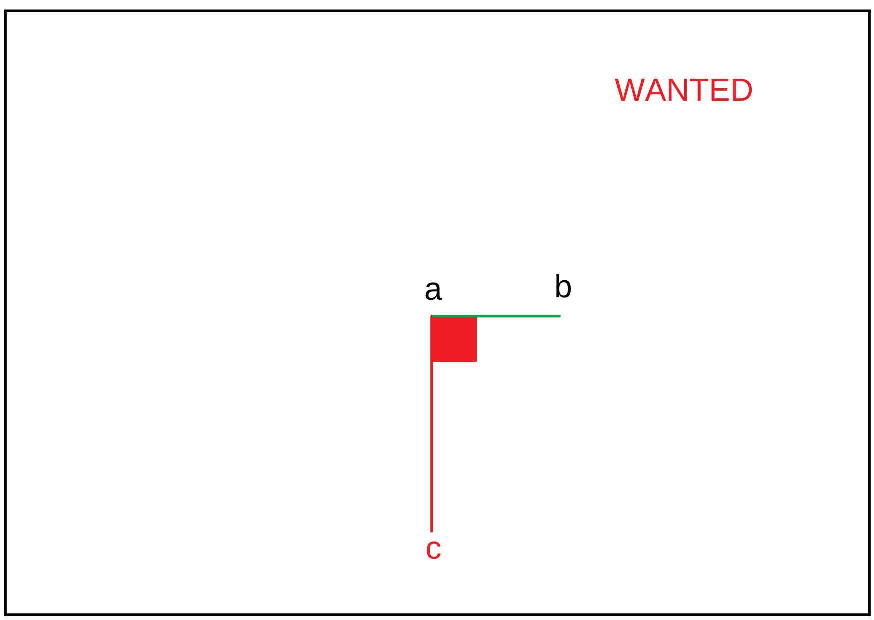
II.11:9. For let the square ABCD be described on AB; [I.46]

GOSUB I.46. We will use the alternate construction, C14B.




I.46:5. Let AC be drawn at right angles to the straight line AB from the point A on it [I.11],

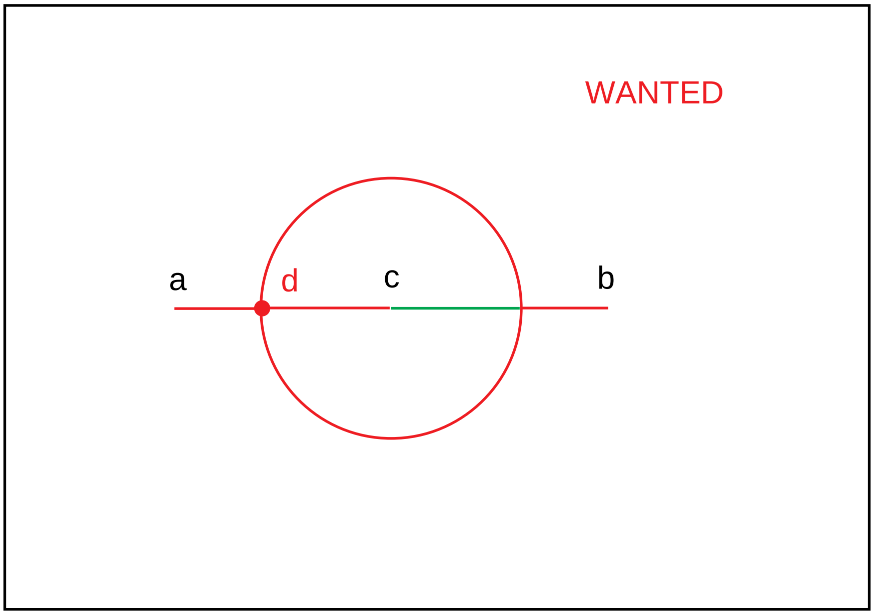
GOSUB I.11.
Extend the line.
Relabel.



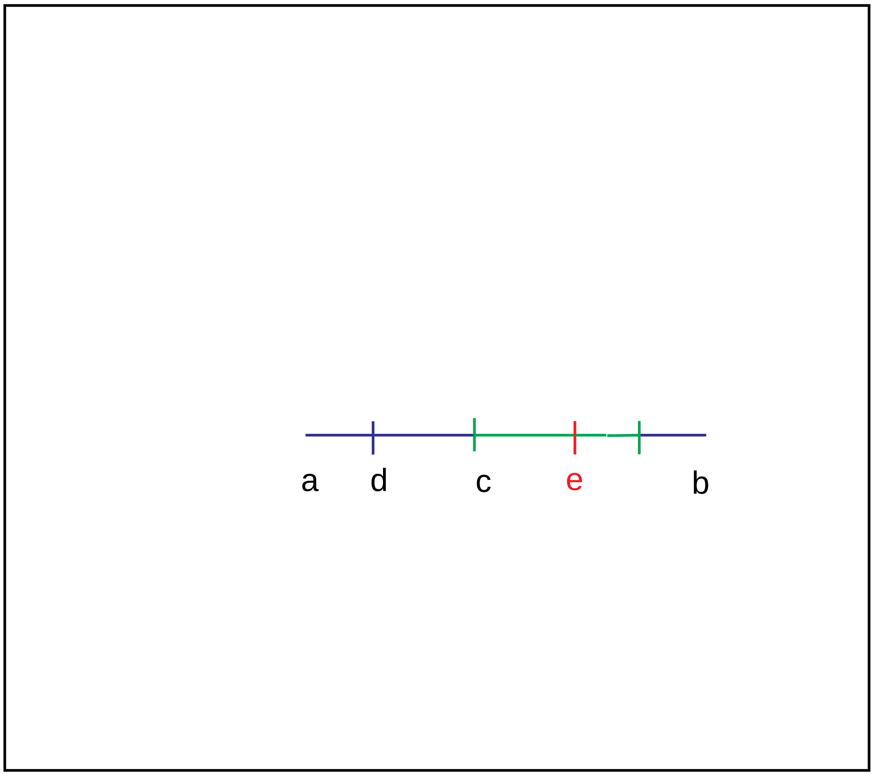
I.11:8. Let a point d be taken at random on ac;
let dc be made equal to ce; [I.3]

*NOTE*: Rather than use dividers in place of I.3 as usual (cf. I.3) it will be useful here to use the compass and draw a circle as shown.

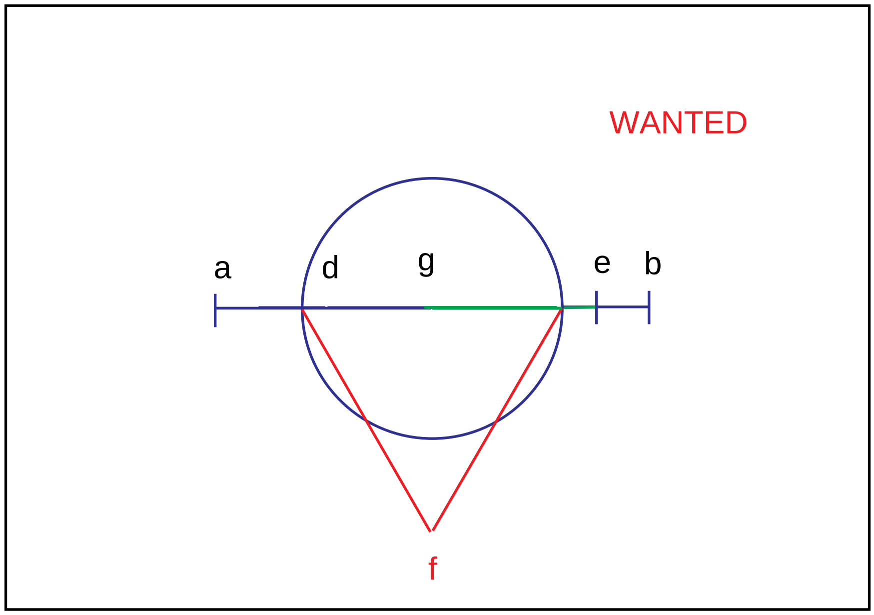
With centre c and distance cb let the circle be described. Let d be the point through which this circle crosses a'd.




I.11:10. let e be made equal to cd;
[I.3] (dividers)

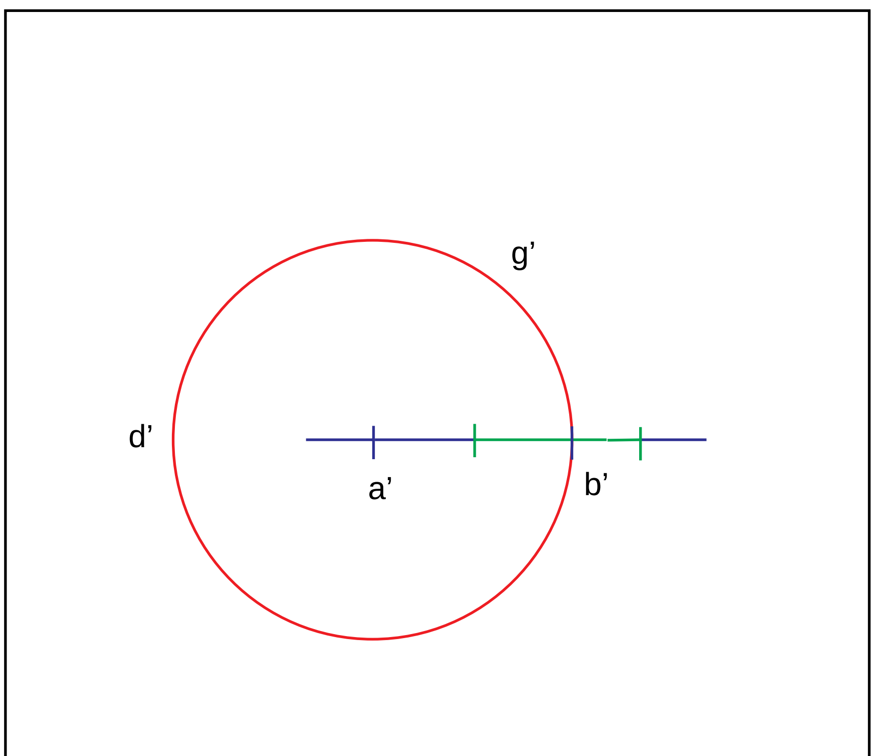


I.11:11. on de let the equilateral triangle fde be constructed, [I.1]

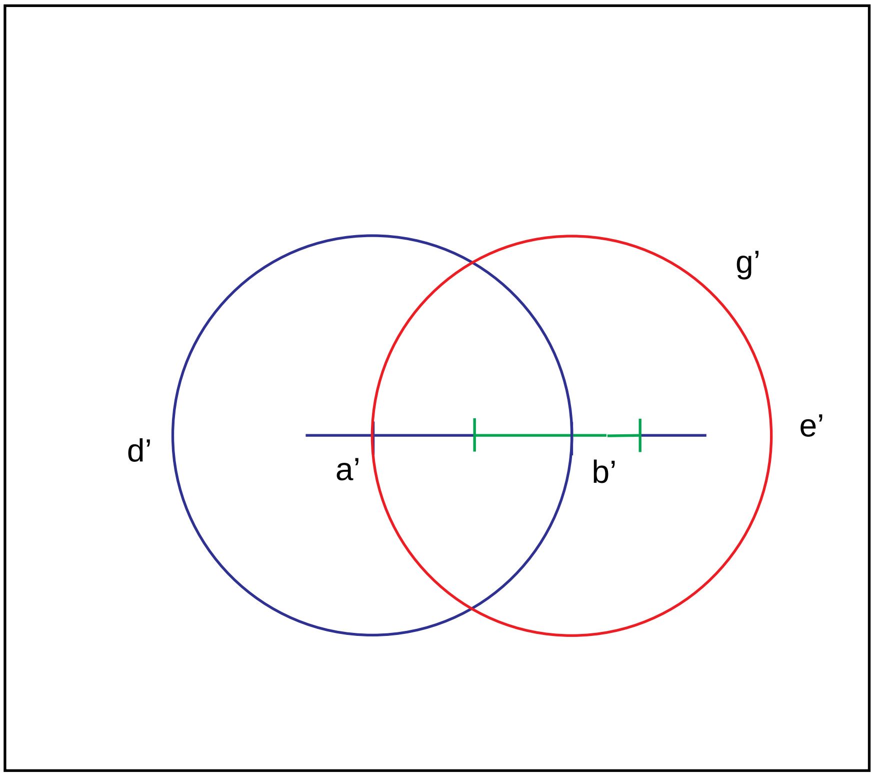



GOSUB I.1.
Relabel.

I.1:7. With centre a’ and distance a’b’ let the circle b’g’d’ be described [Post 3]

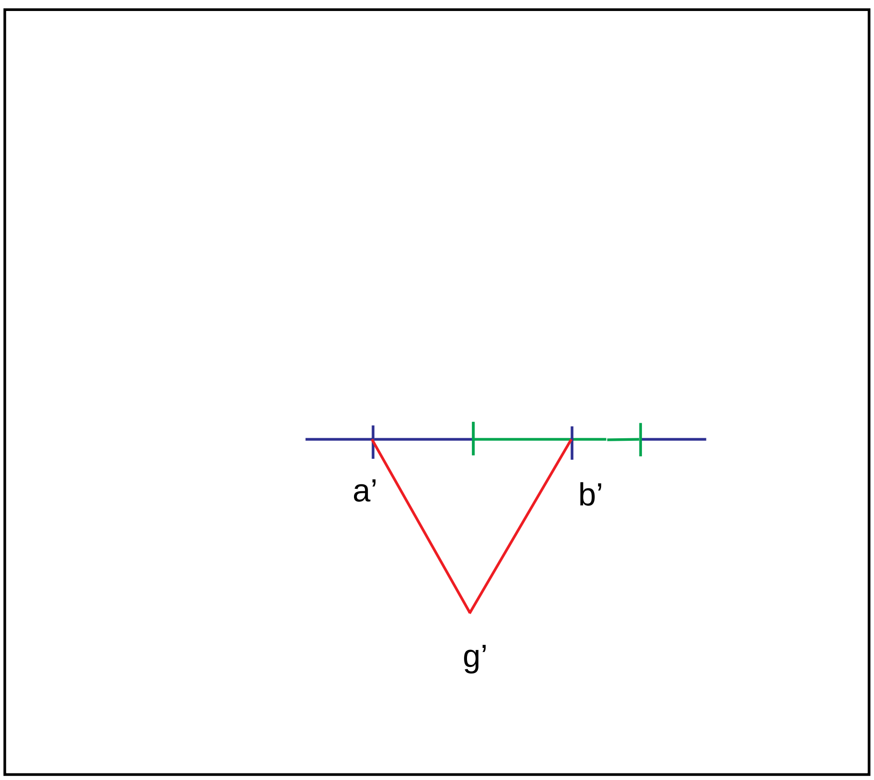


I.1:10. again, with centre b’ and distance b’a’ let the circlea’g’e’ be described; [Post 3]

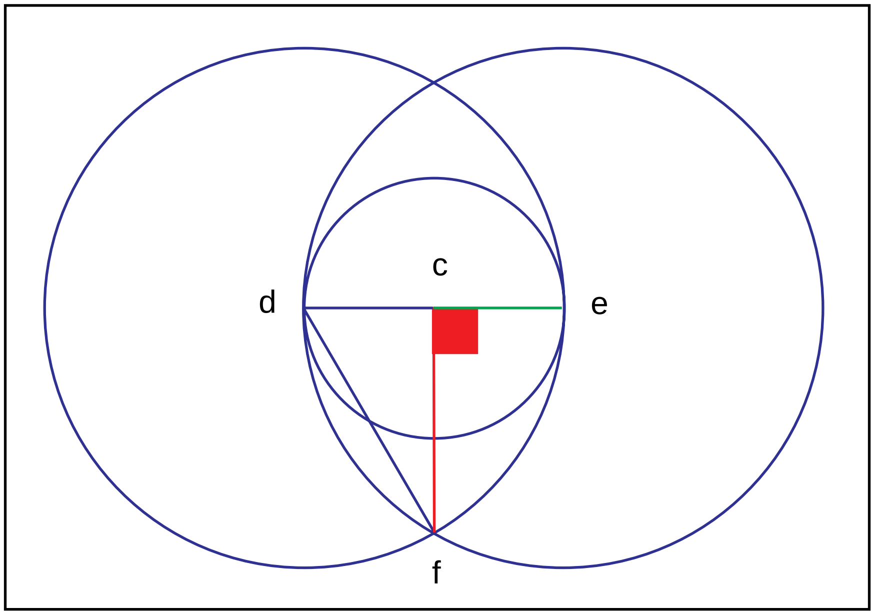



I.1:13. and from the point g’, in which the circles cut one another, [we choose the lower one] to the points a’, b’, let the straight lines g’a’, g’b’ be joined. [Post. 1]

Cleanup. (Retain the circle of step 1). RETURN to I.11 at line 11.
Relabel.

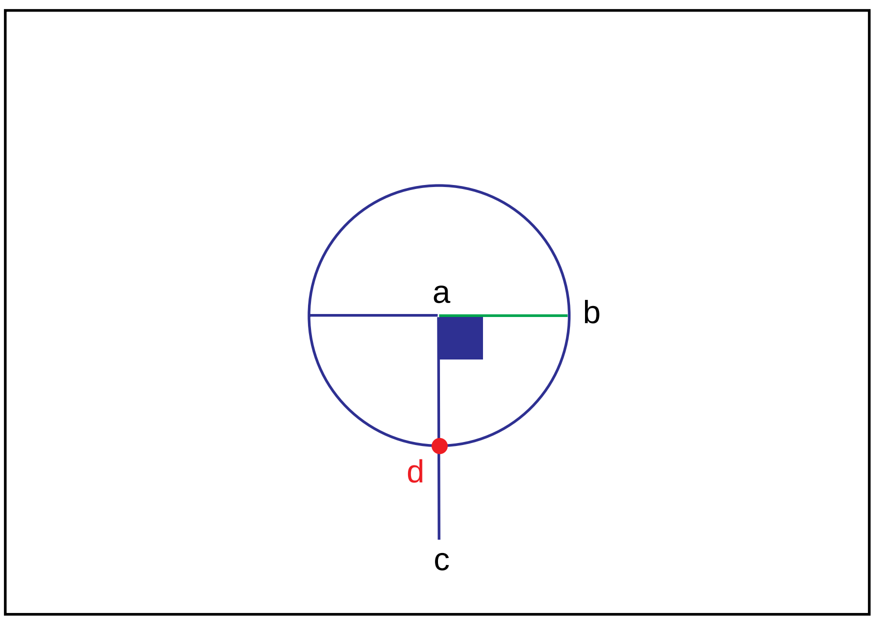


I.11:13. and let fc be joined;
I say that the straight line fc has been drawn at right angles to the given straight line de from c the given point on it.



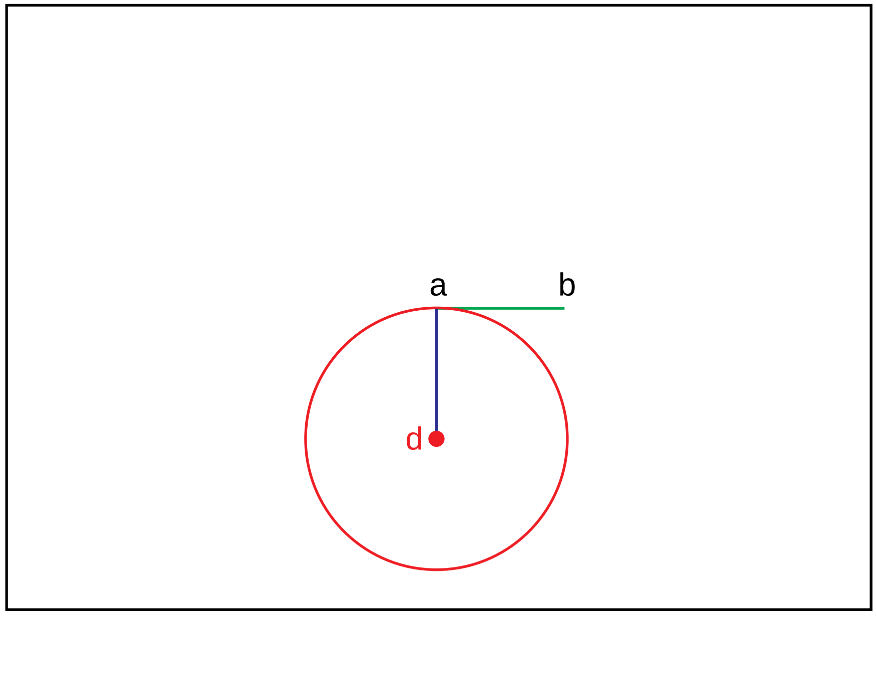

Cleanup. Retain the circle from step 1.
RETURN to I.46 at
line 5.

I.46:11. and let ad be made equal to ab;

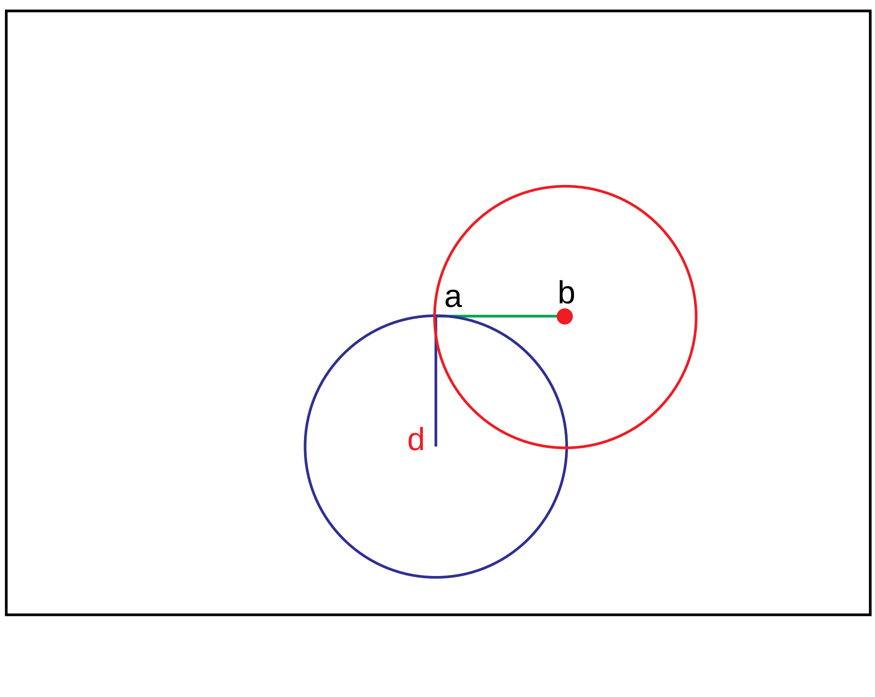



We may now save steps by following C#14B to complete the square.

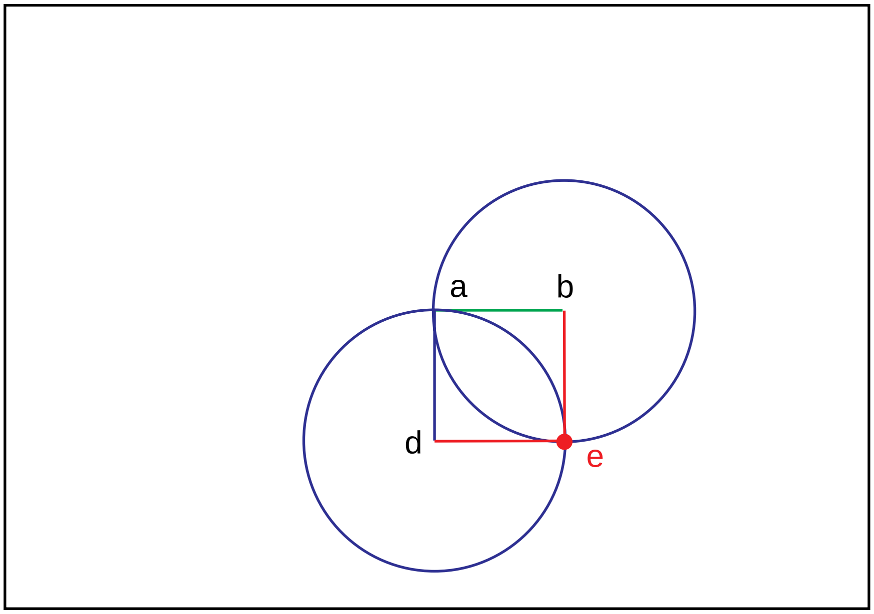
Swing da around d.




Swing ba around b.



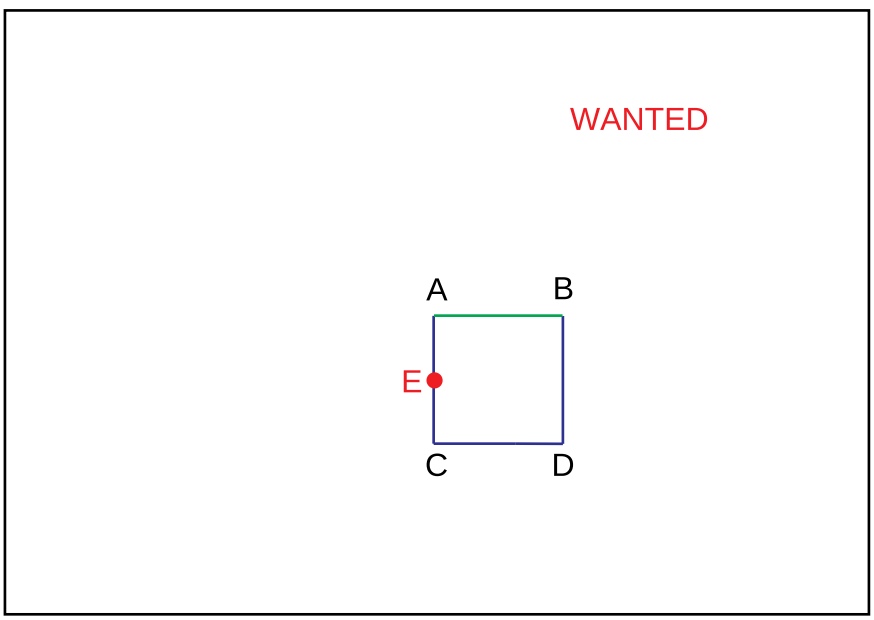
Connect the crossing point, e, to the centers.




Cleanup.
RETURN to I.11 at line 9.

Relabel.

I.11:11. let AC be bisected at the point E, ([I.10])

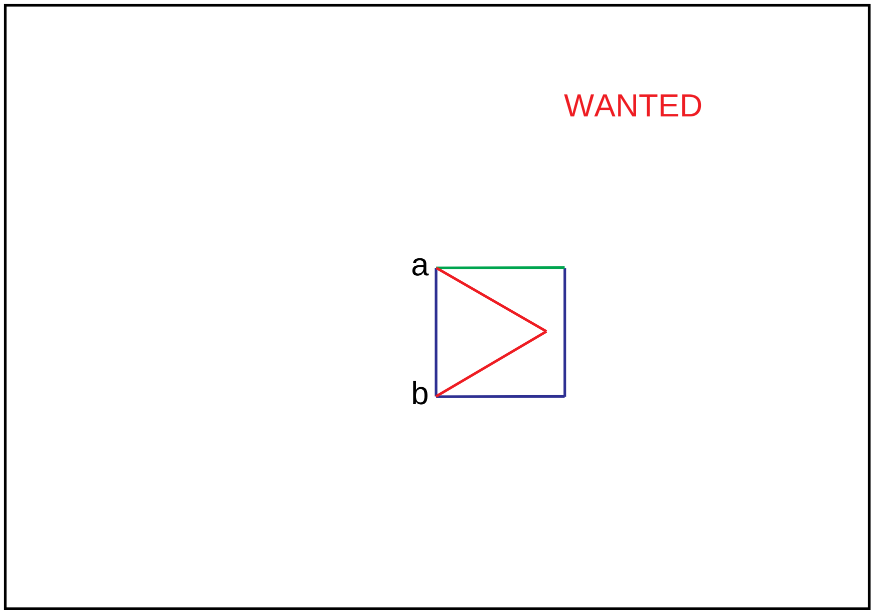



GOSUB I.10. Relabel.

I.10:2. Let ab be the given finite straight line.

I.10:4 Let the equilateral triangle abc be constructed on it, [I.1] (For economy, we will follow the shortcut, C#5B.)

GOSUB I.1 (Vesica Pisces)

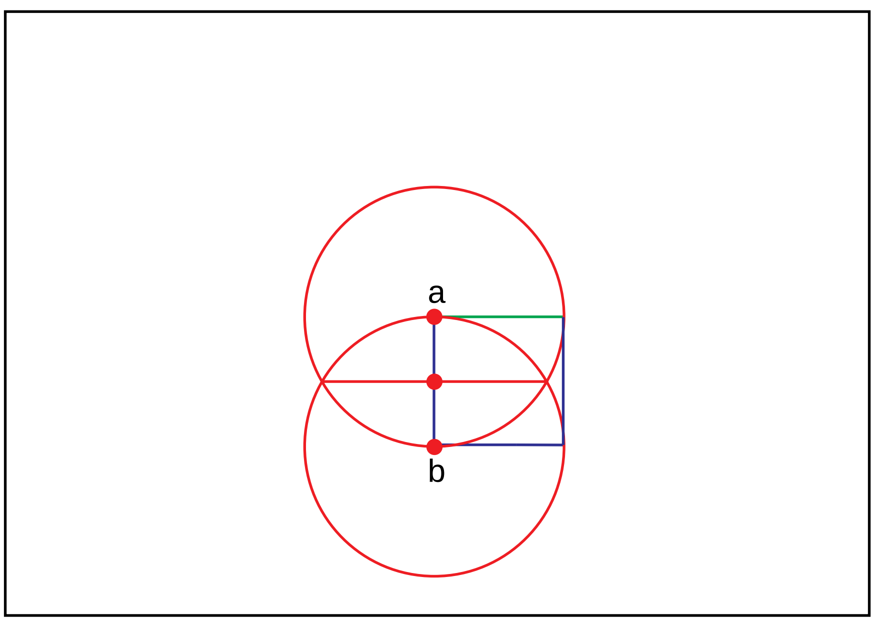



I.1:7. With centre a and distance ab let the circle be described; (Revive step 6))

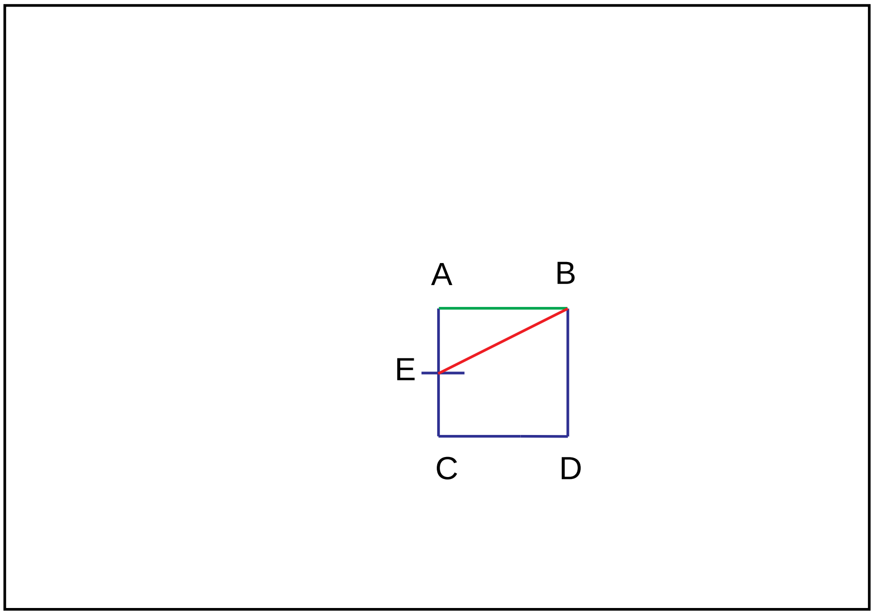
I.1:10. again, with centre b and distance ba let the circle be described;
(Revive step 9))

Connect the crossing points. Mark the point where this line crosses ab.

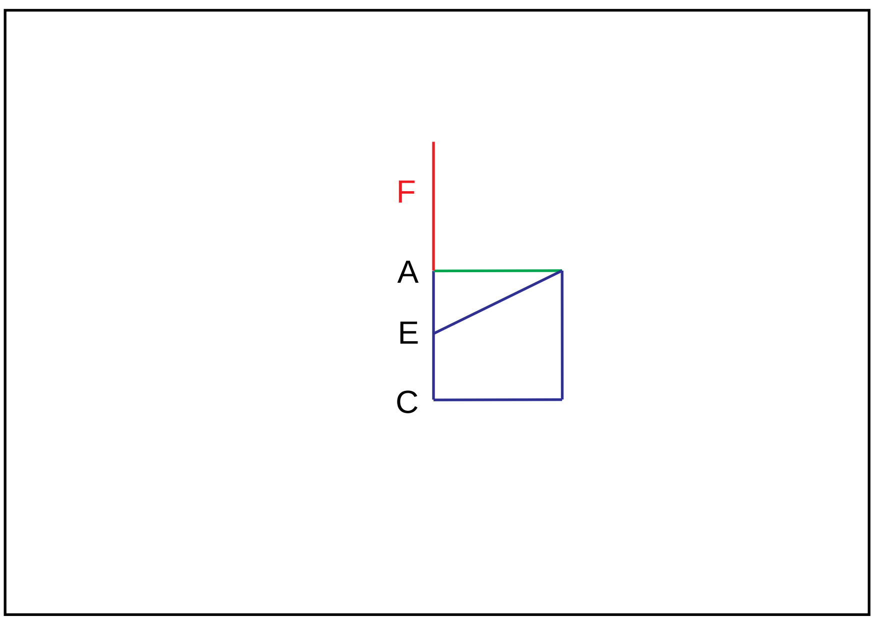
Cleanup and RETURN to II.11 at line 11.



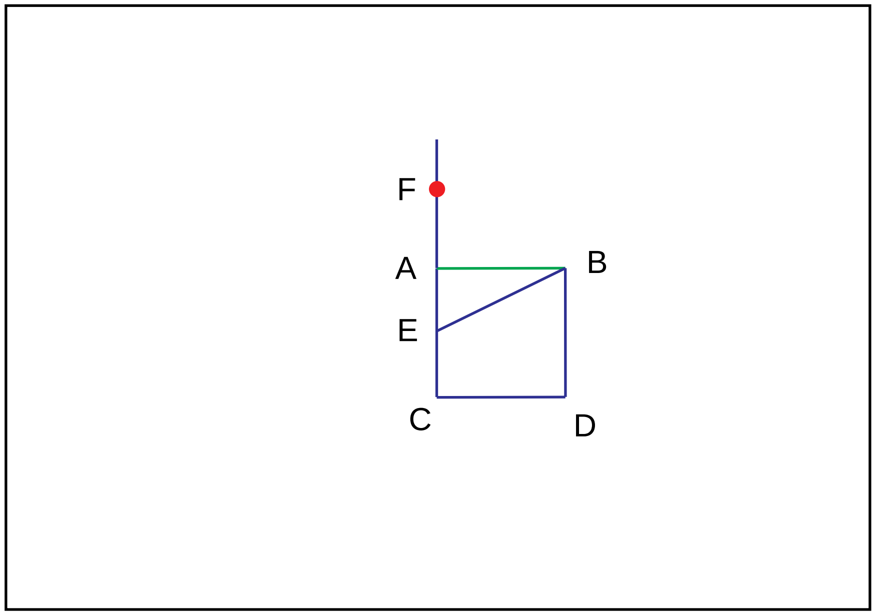
I.11:11. and let BE be joined;




I.11:12. let CA be drawn through to F, (this point is not actually located until the next step.)

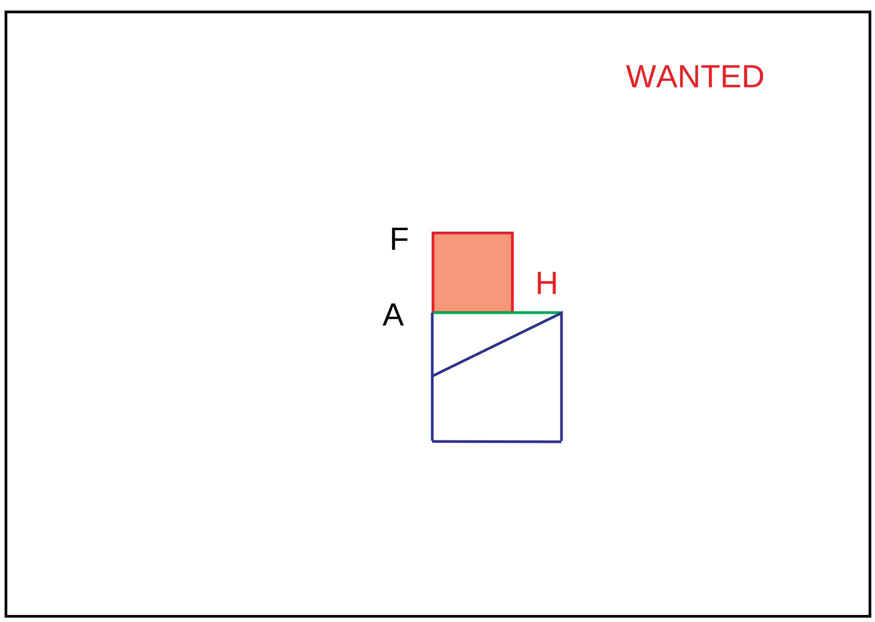


I.11:12. and let EF be made equal to BE; ([I.3], or dividers)



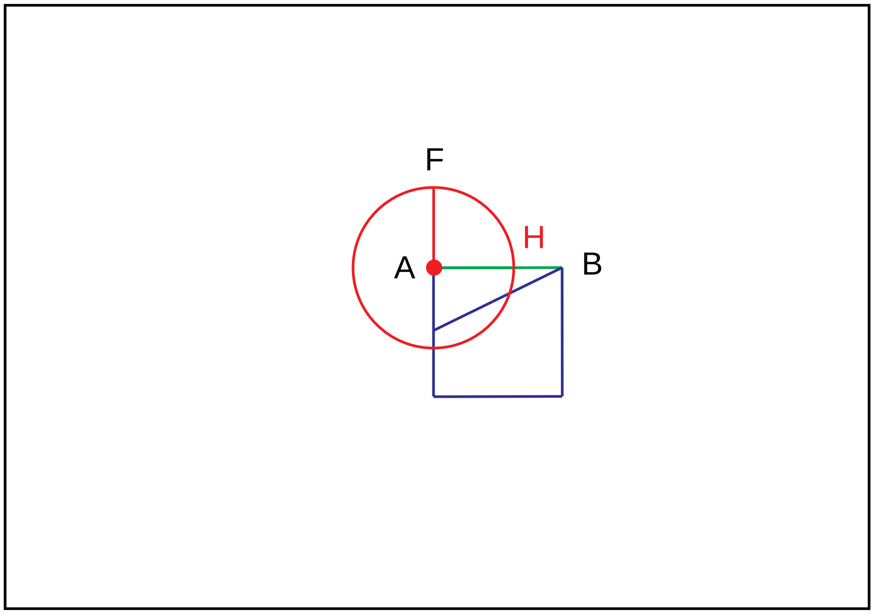

I.11:14. let the square FH be described on AF,

GOSUB I.46
(We will use the variation C#14B: swing three arcs with the same compass.)

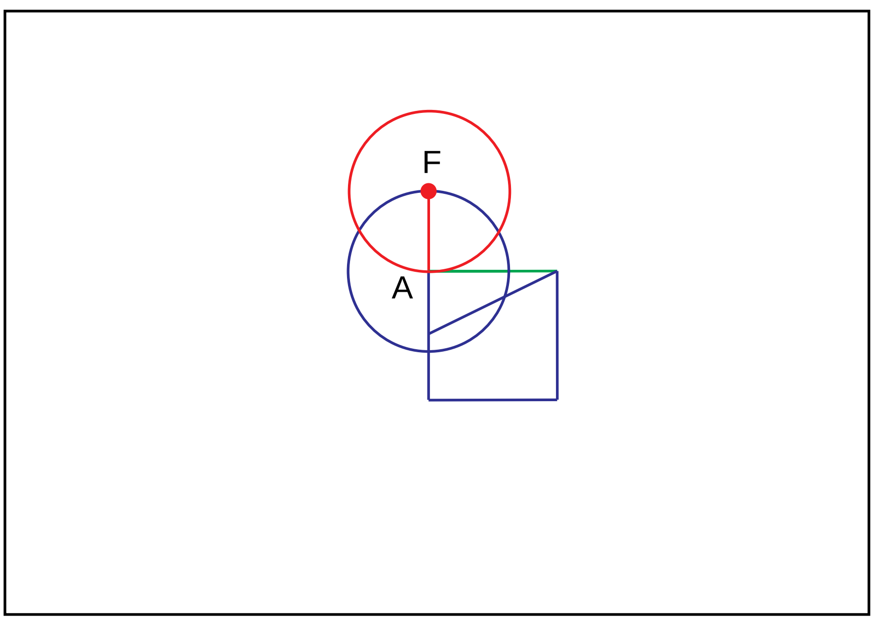


Swing from the corner, A. Locate the point H on AB so AH is equal to AF. (Dividers.)

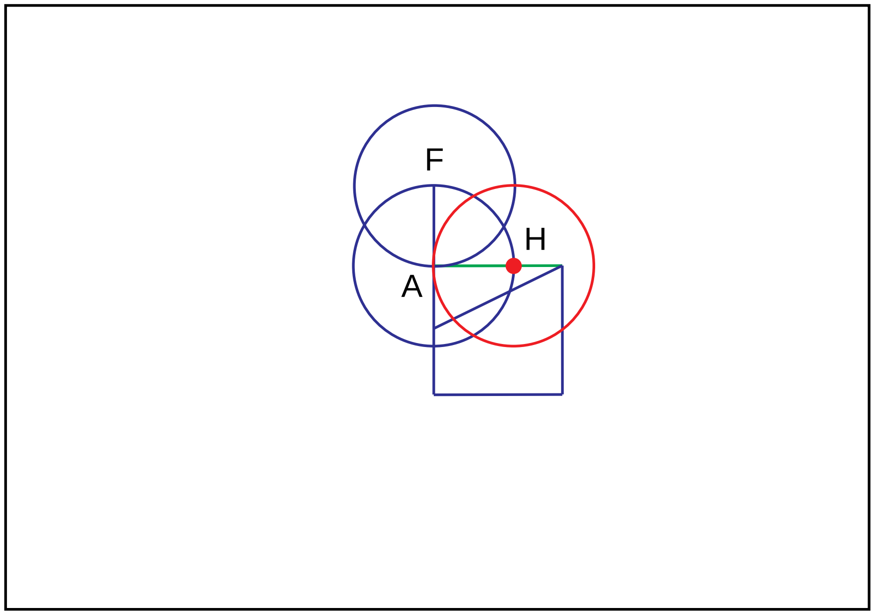
*NOTE*: With this point we are finished with the first instruction: To cut a given straight line. This is the golden section in 14 steps. We include the next steps, formally part of the proof machinery, to see what we have done.




Swing from the old end, F.

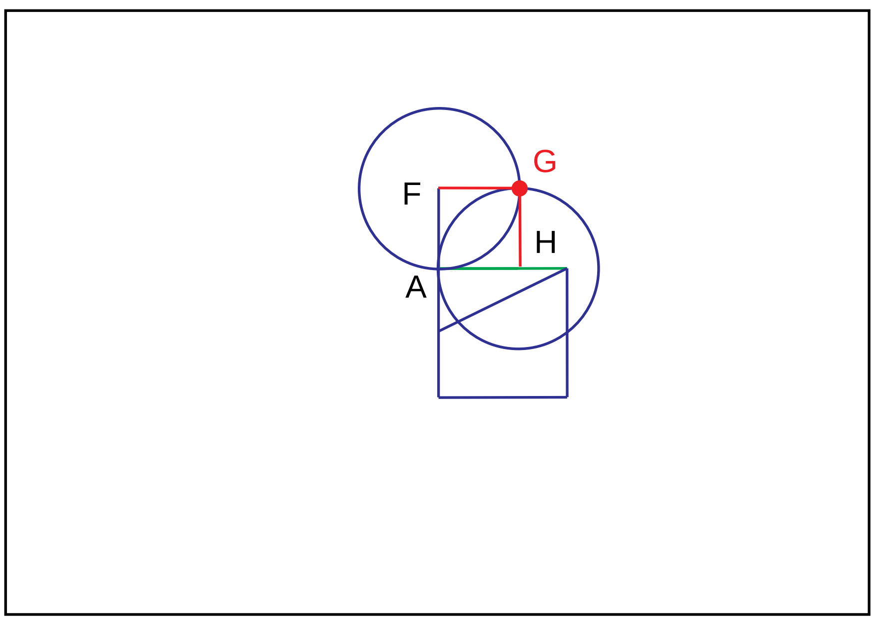


Swing from the new end, H.

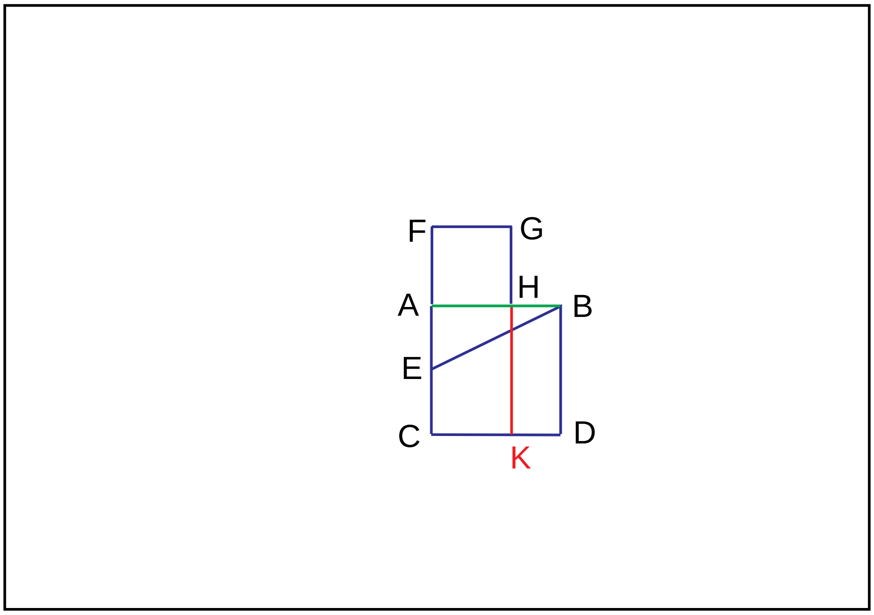



Connect the crossing point, G, to the two ends, F and H.

Cleanup.
RETURN to I.11 at line 14.



I.11:14. and let GH be drawn through to K.




*NOTE*: These last 4 steps belong to the machinery. The golden section was done at step 14. Compare VI.30.

Q.E.F.