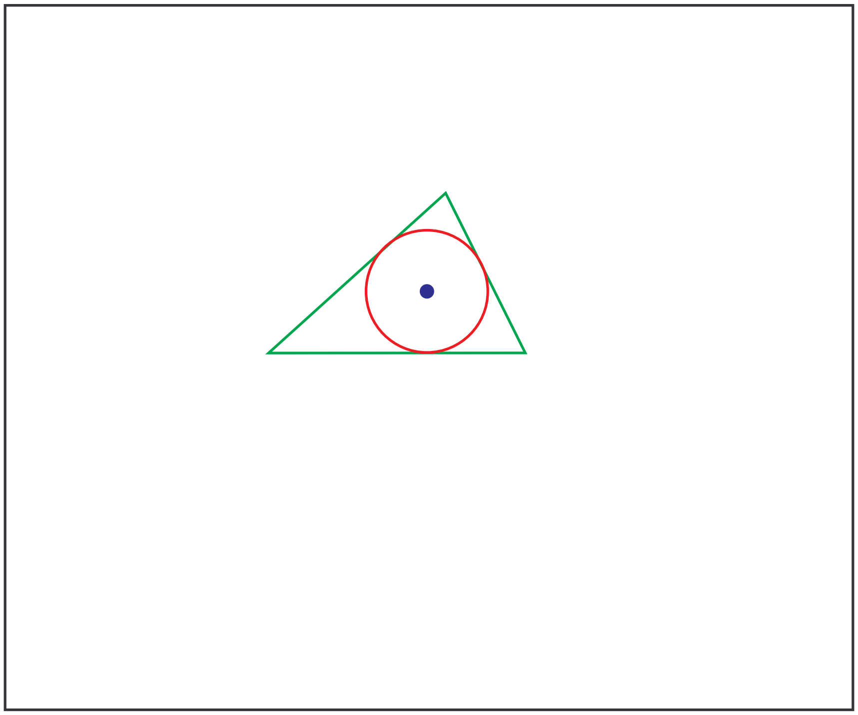
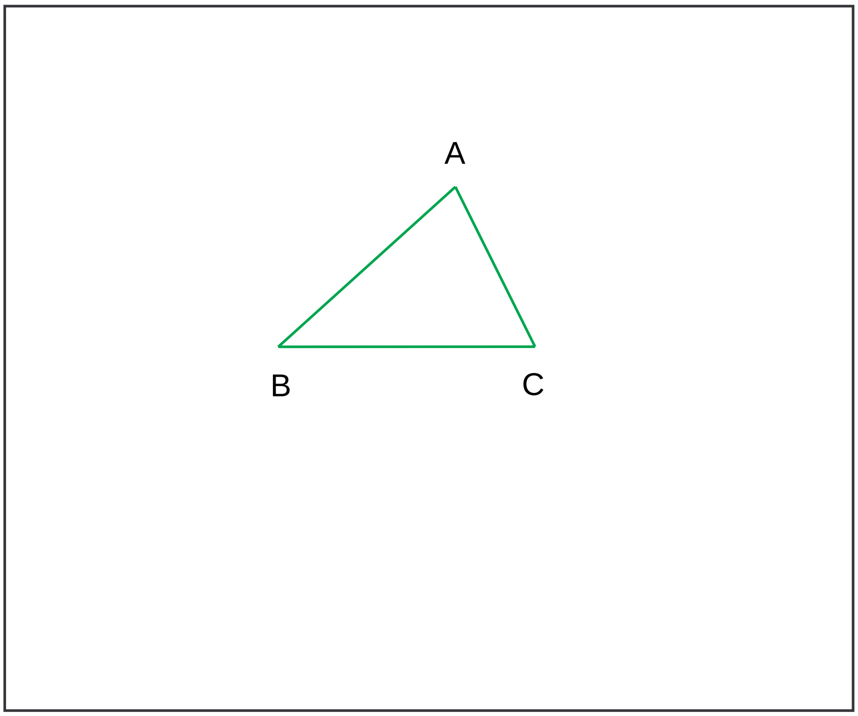
Construction 26: Book IV, Proposition 4

.

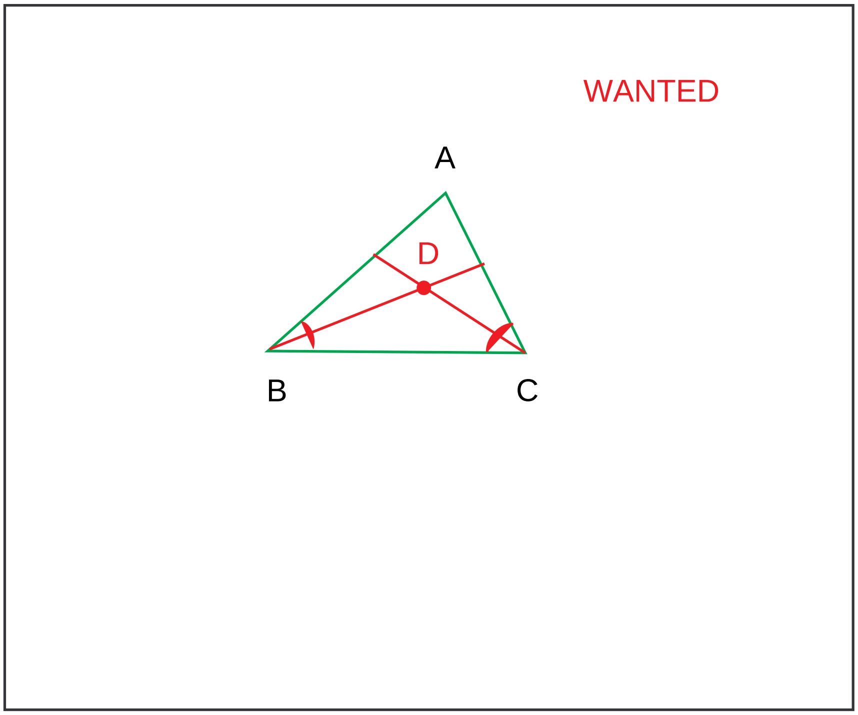
In a given triangle to inscribe a circle.



IV.4:2. Let ABC be the given triangle;

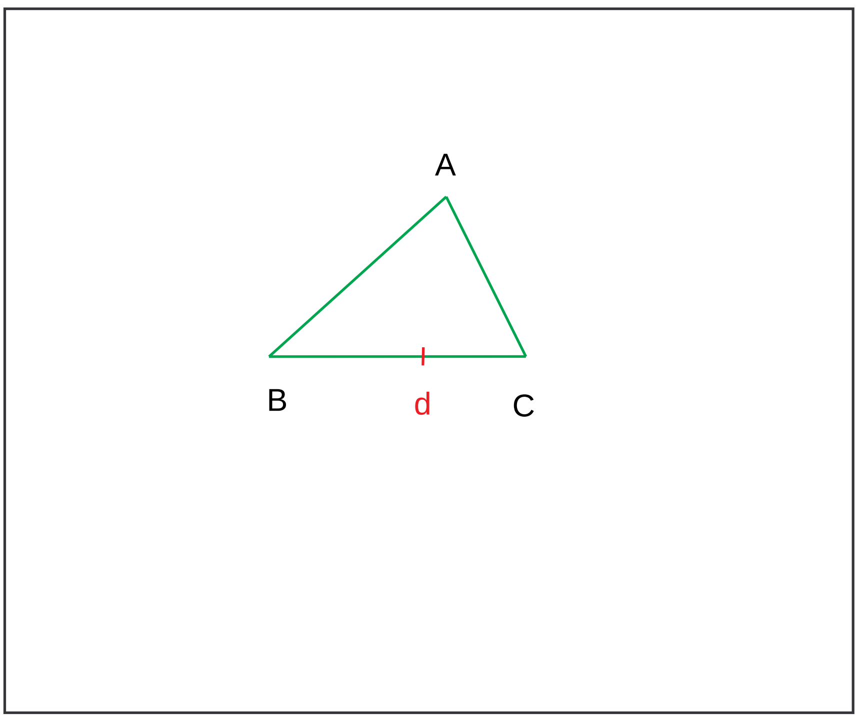


IV.4:4. Let the angles ABC, ACB be bisected by the straight lines BD, CD [I.9], and let these meet one another at the point D;

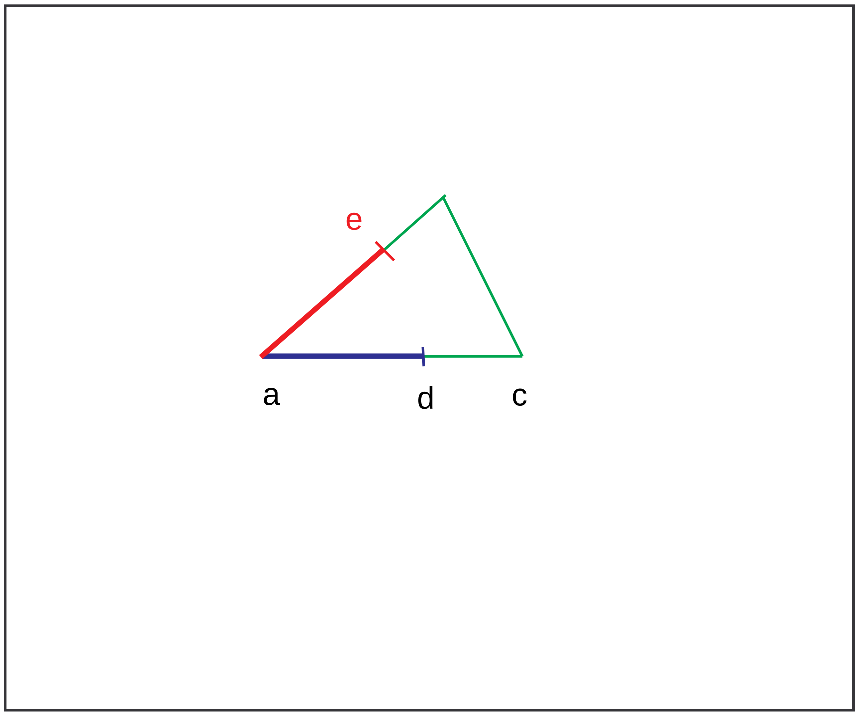


GOSUB I.9 (C#4) for the angle ABC.

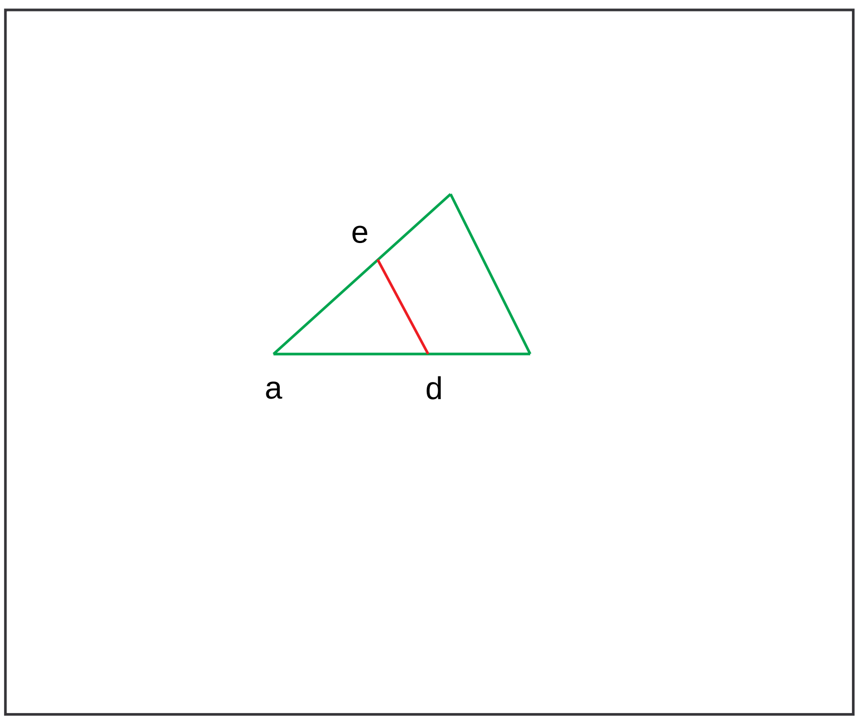
I.9:4. Let a point d be taken at random on ac;



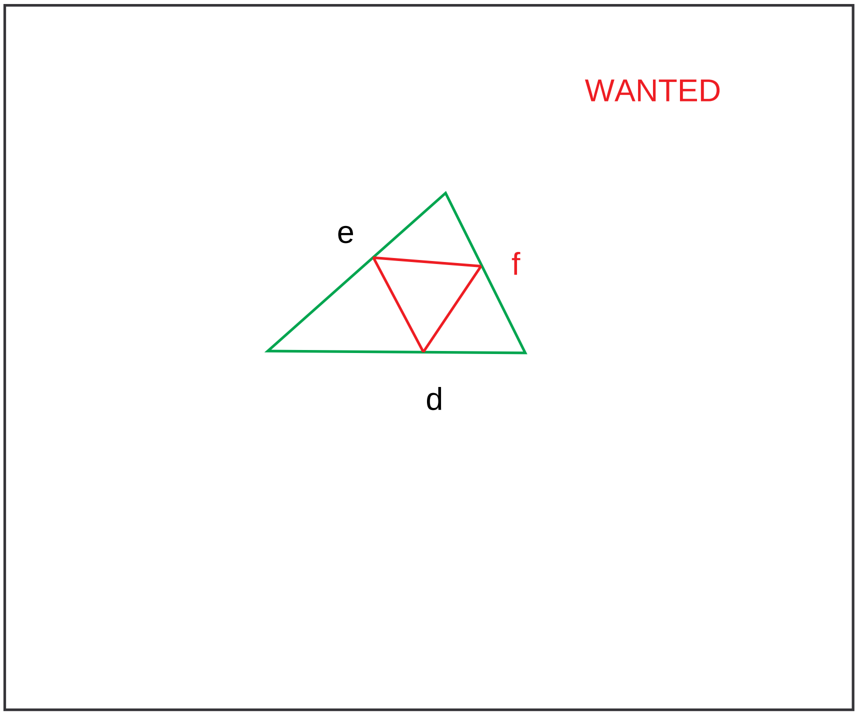
I.9:5. let ae be cut off from ac equal to ad; [I.3]
(use the rope)



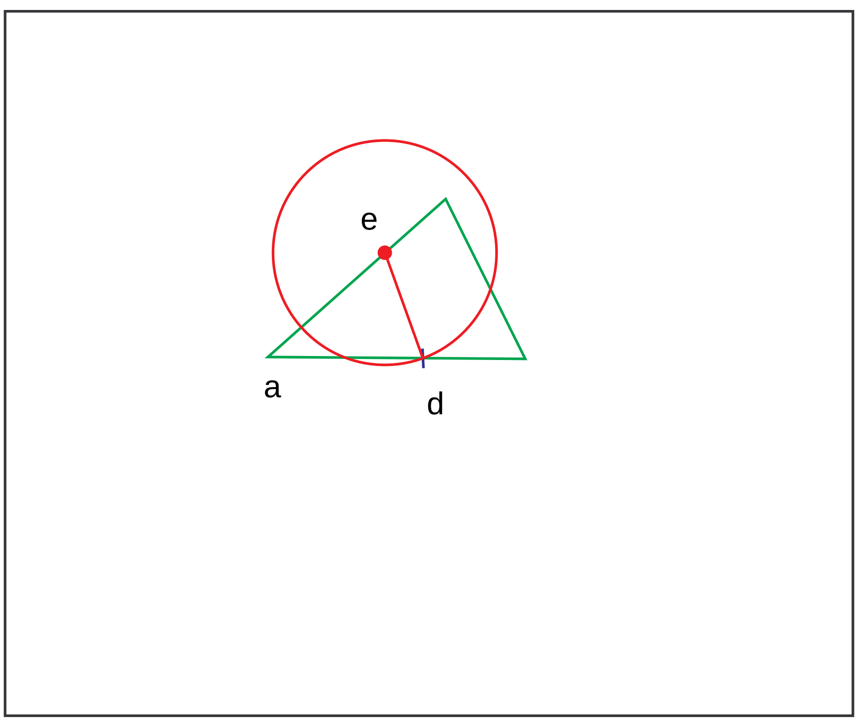
I.9:6. let de be joined,



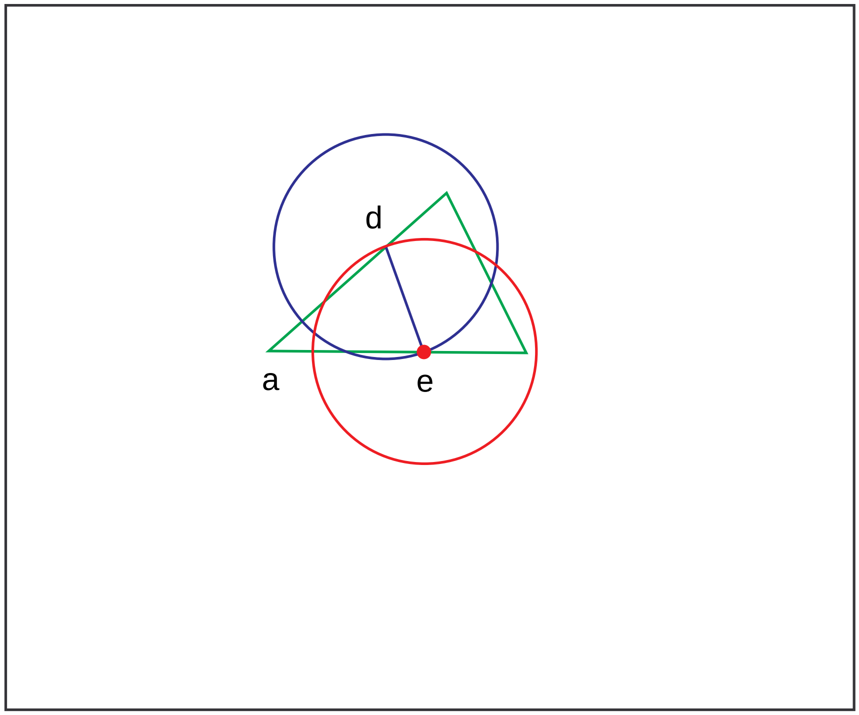
I.9:6. and on de let the equilateral triangle def be constructed; ([I.1])



GOSUB I.1.
Swing de around e.

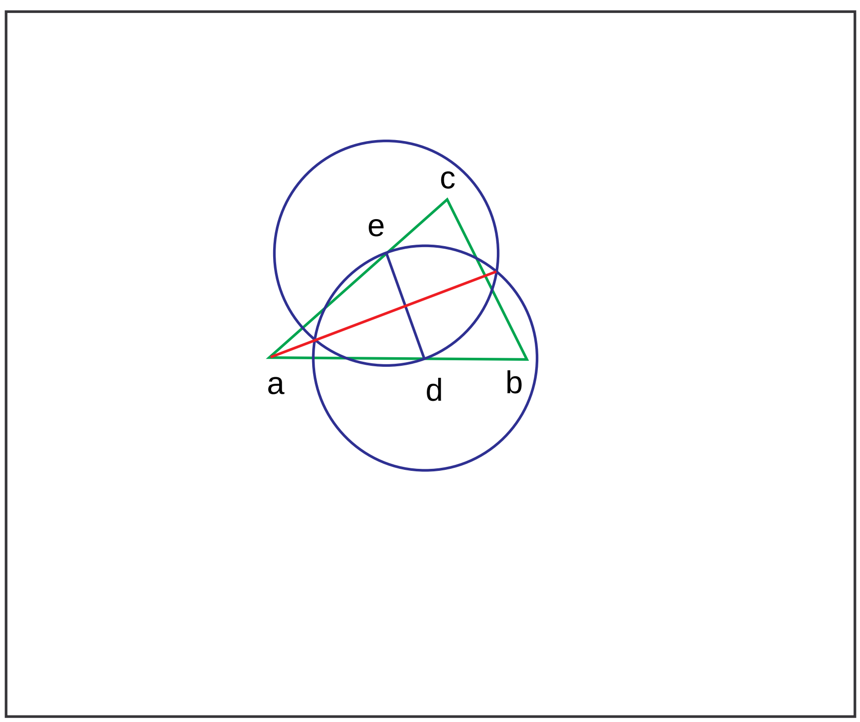


Swing ed around d.



Now we have the Vesica Pisces,
and we abandon I.1.

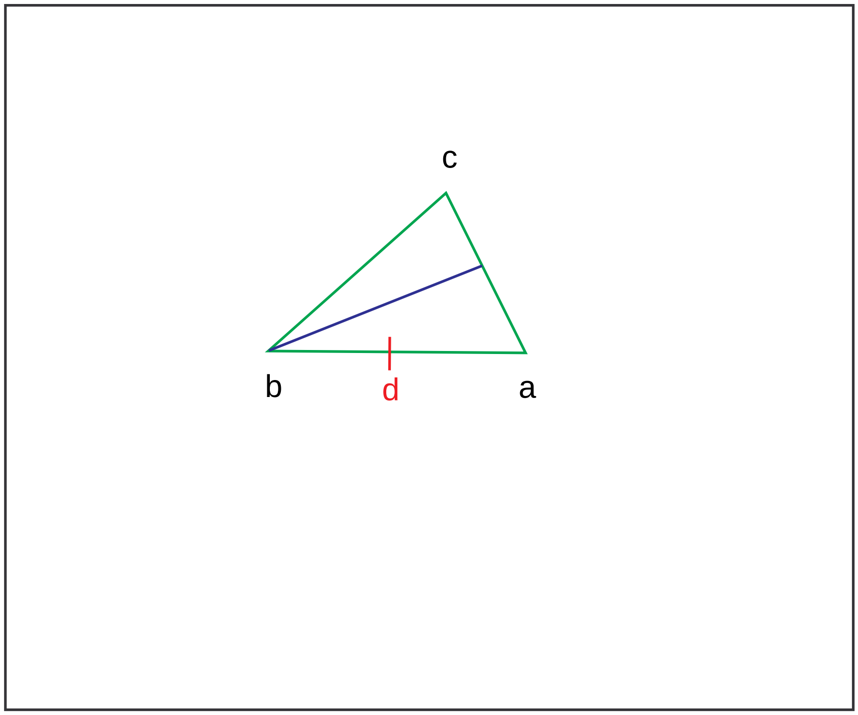
Connect the crossing points
through to a and to the line bc.



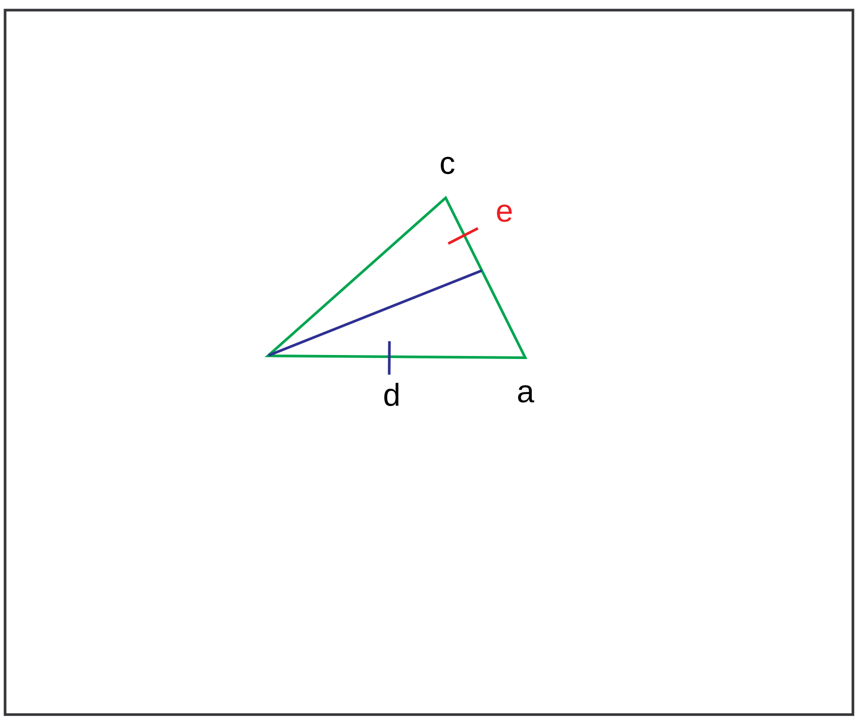
Cleanup.
RETURN to IV.4 at line 4.

GOSUB I.1 again.
Relabel, for angle ACB.

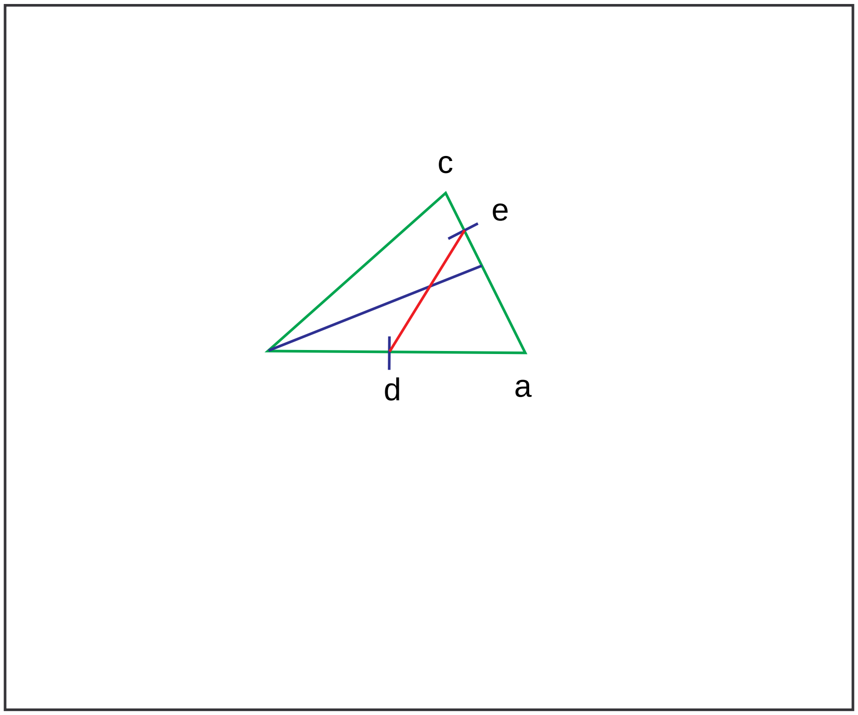
I.9:4. Let a point d be taken at random on ab;



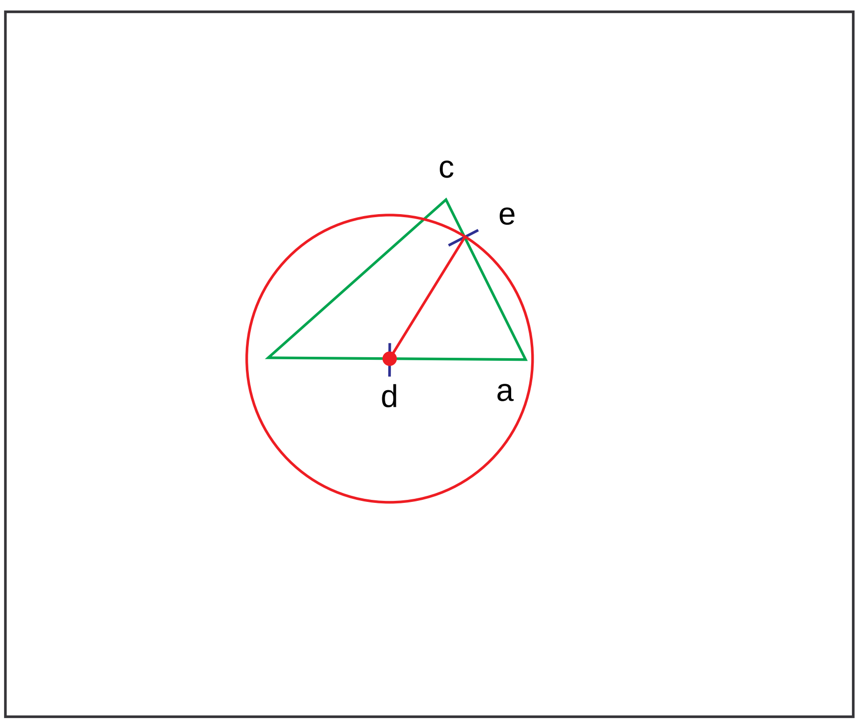
I.9:5. let ae be cut off from ac equal to ad; (rope)



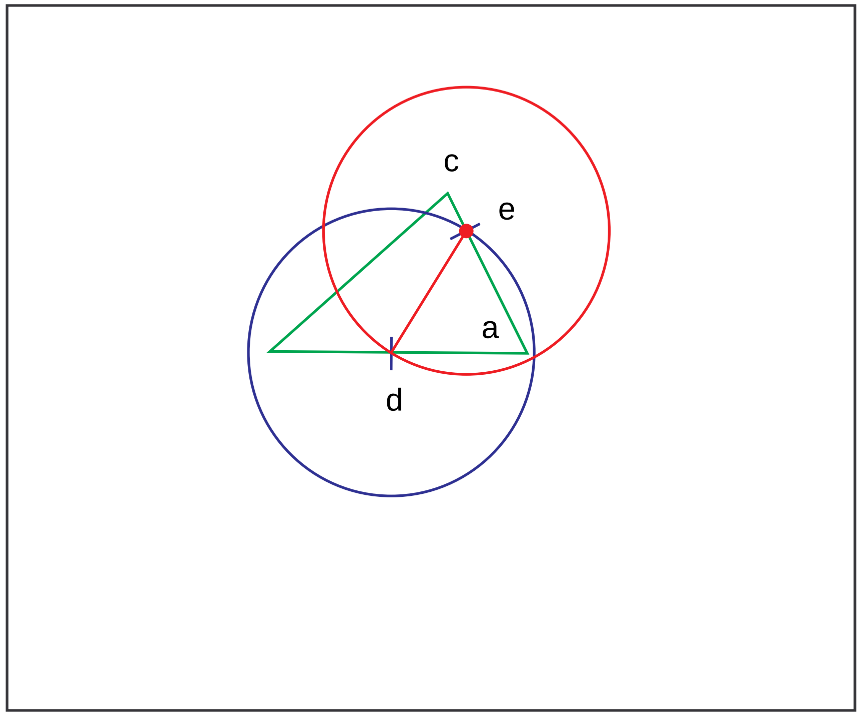
I.9:6. let de be joined,



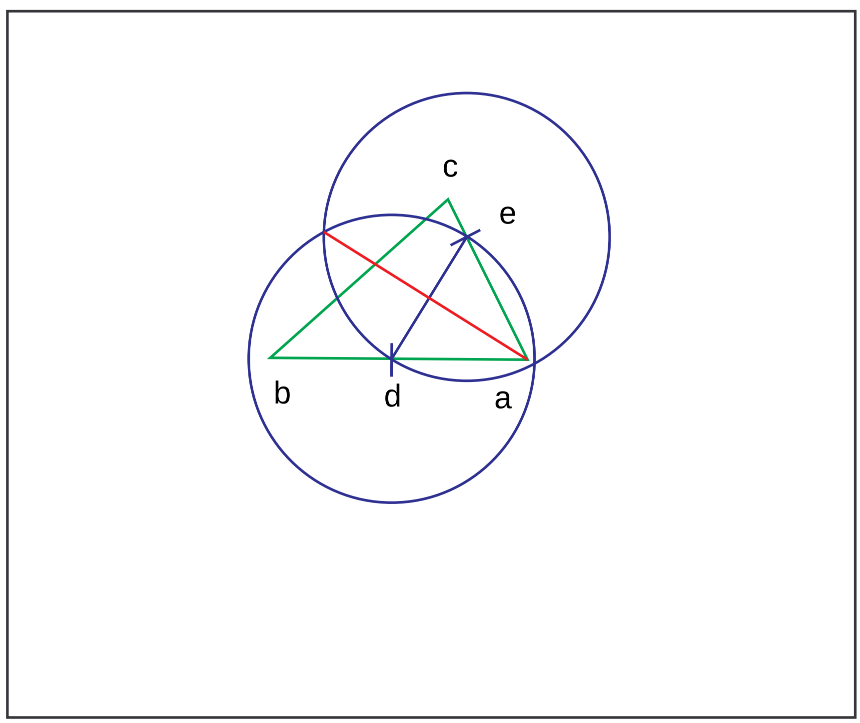
Swing de around d.



Swing ed around e.



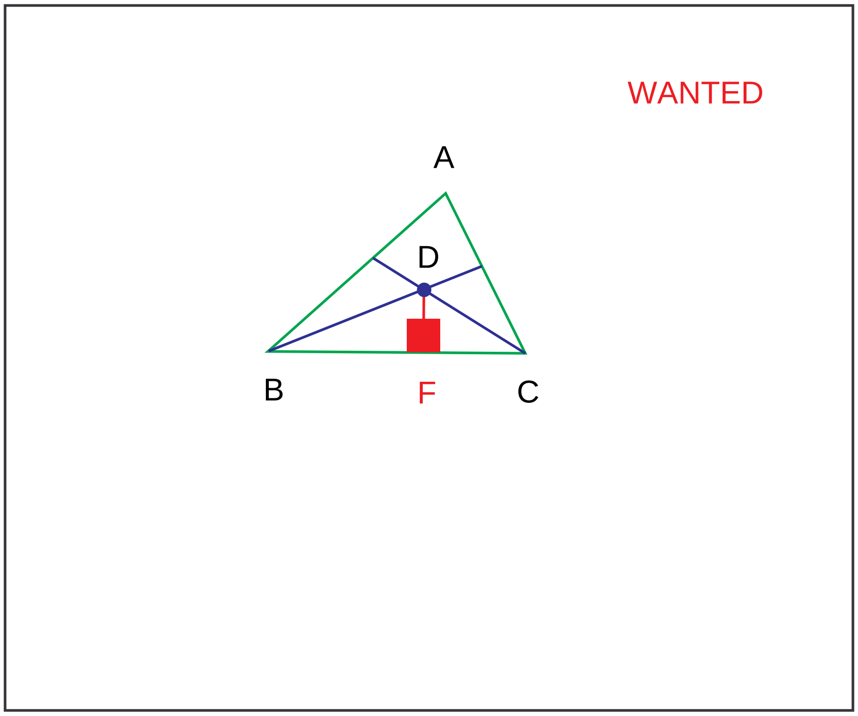
Connect the crossing points through to a.



Cleanup.
Relabel

RETURN to IV.4 at line 4. The two bisecting lines meet at D.

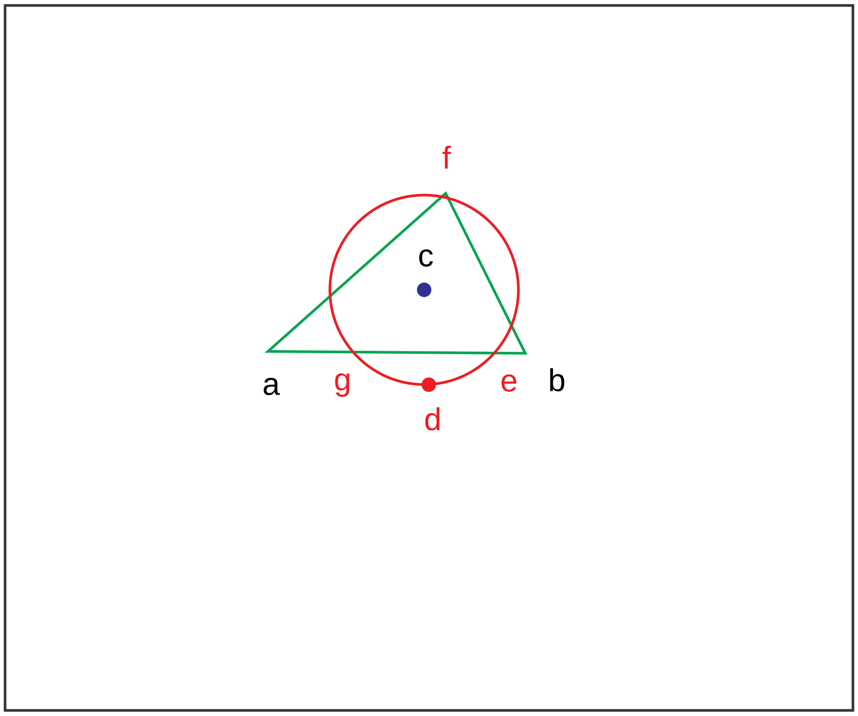
IV.4:8. from D let DE, DF, DG be drawn perpendicular to the straight lines AB, BC, CA. ([I.12])



Actually, we need only one of these for the construction, as opposed to proof. We will draw DF

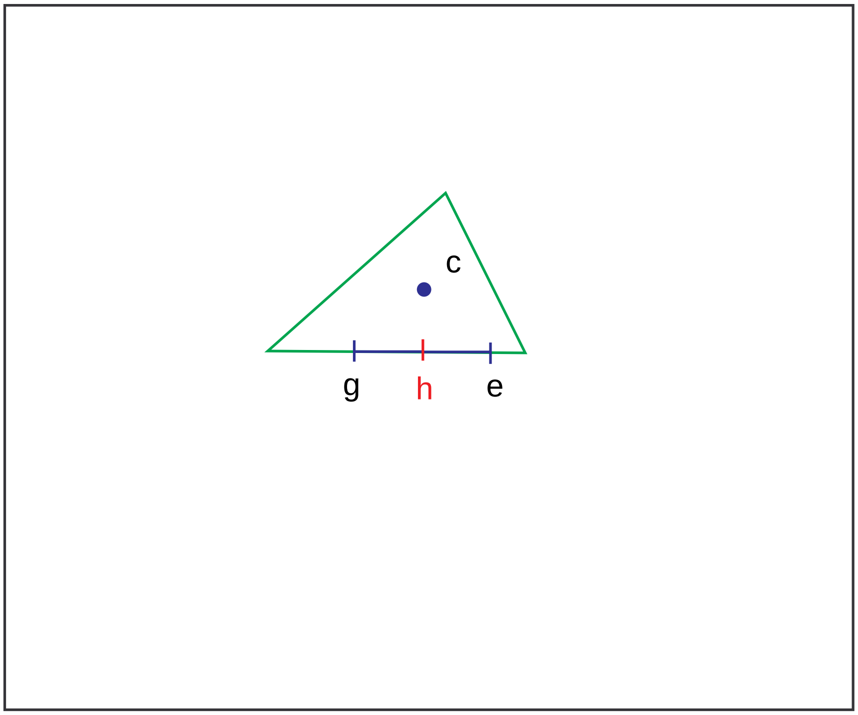
GOSUB I.12.
Relabel.

I.12:9. For let a point d be taken at random on the other side of the straight line ab, and with centre c and distance cd let the circle efg be described; [Post. 3]



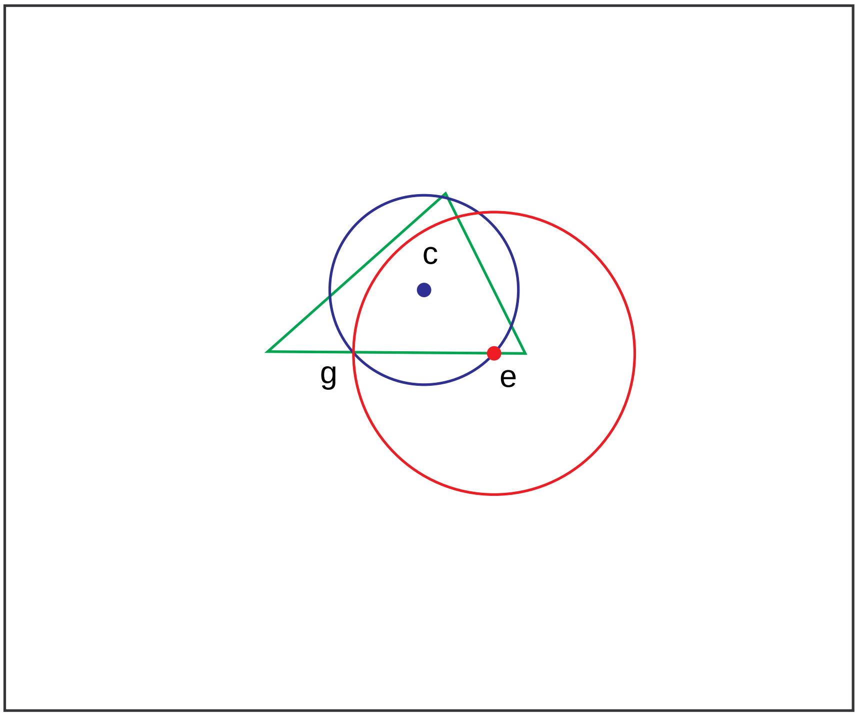
(The circle cuts ab in the points e and g).

I.12:14. let the straight line eg be bisected at h, [I.10]

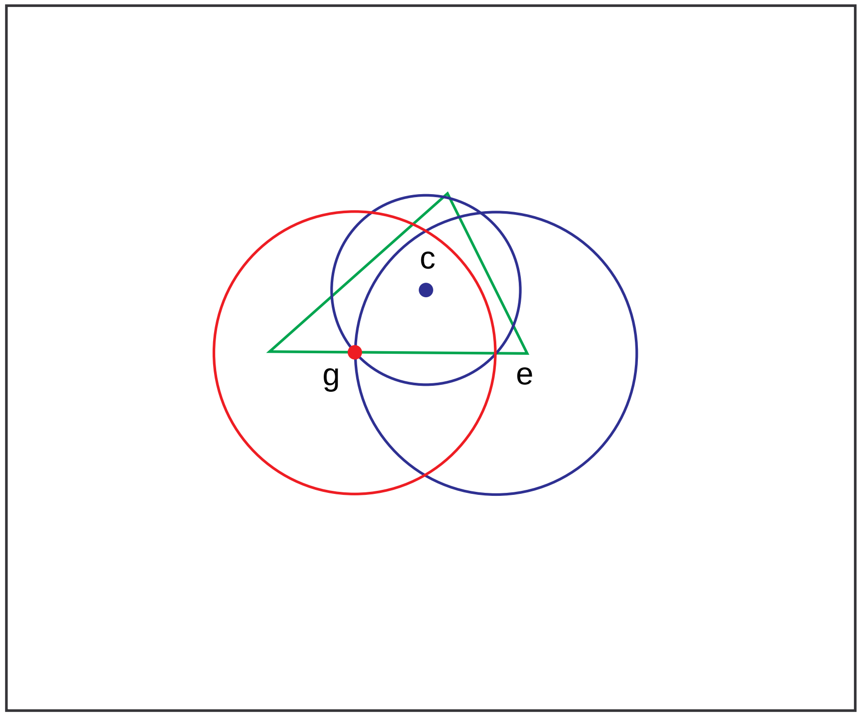


GOSUB I.10

Swing eg around e.

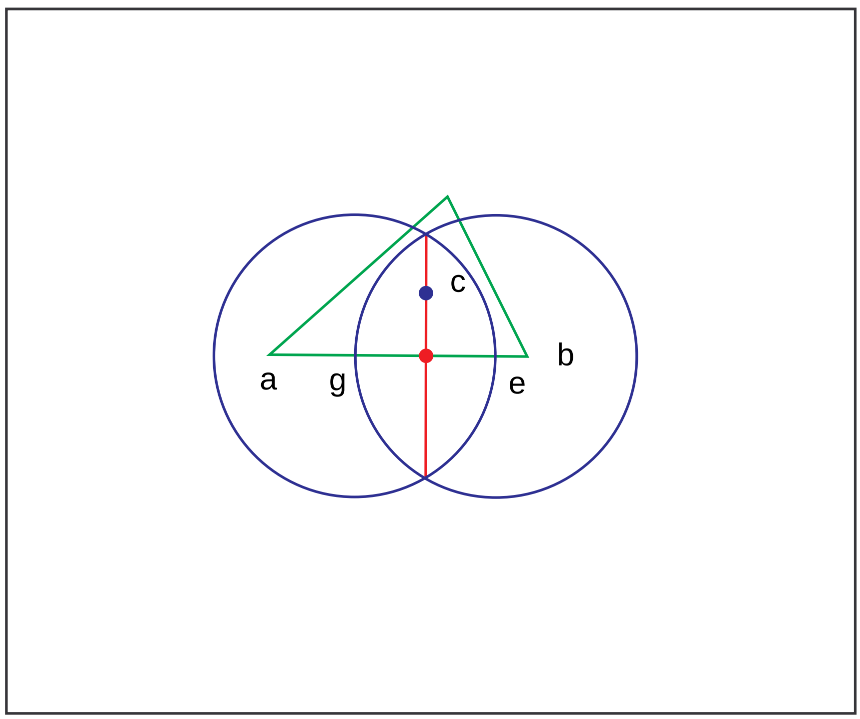


Swing ge around g.



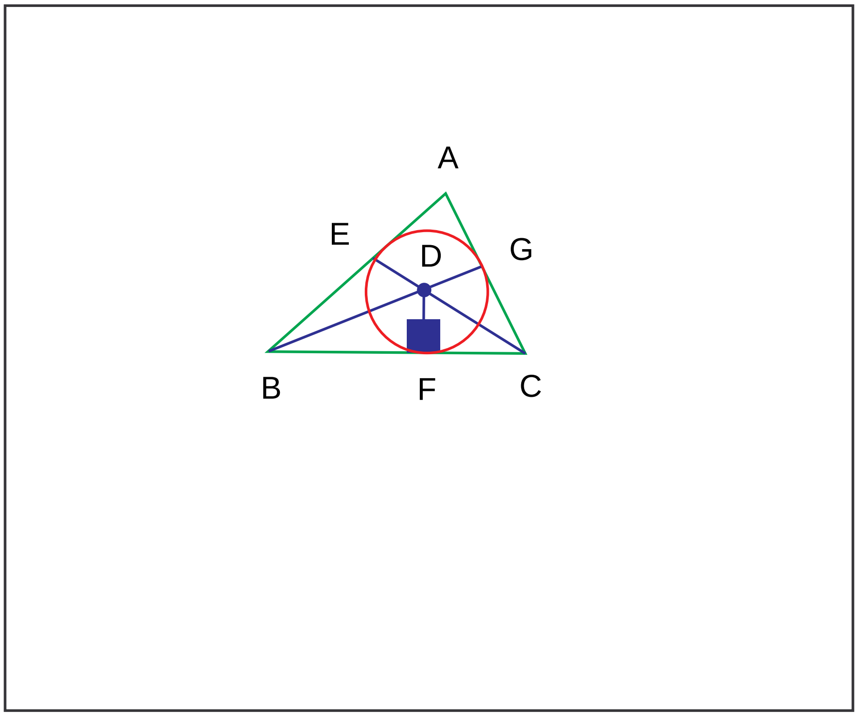
Connect the crossing points to locate h on ab.

Cleanup.
RETURN to I.12 at line 14.
RETURN to IV.4 at line 8.
Relabel.



IV.4:34. Therefore the circle described with centre D and distance one of the straight lines DE, DF, DG will not cut the straight lines AB, BC, CA; Therefore it will touch them, and will be the circle inscribed in the triangle ABC. [IV. Def. 5]

Let it be inscribed, as FGE.



Q.E.F.