Construction 32: Book IV, Proposition 10

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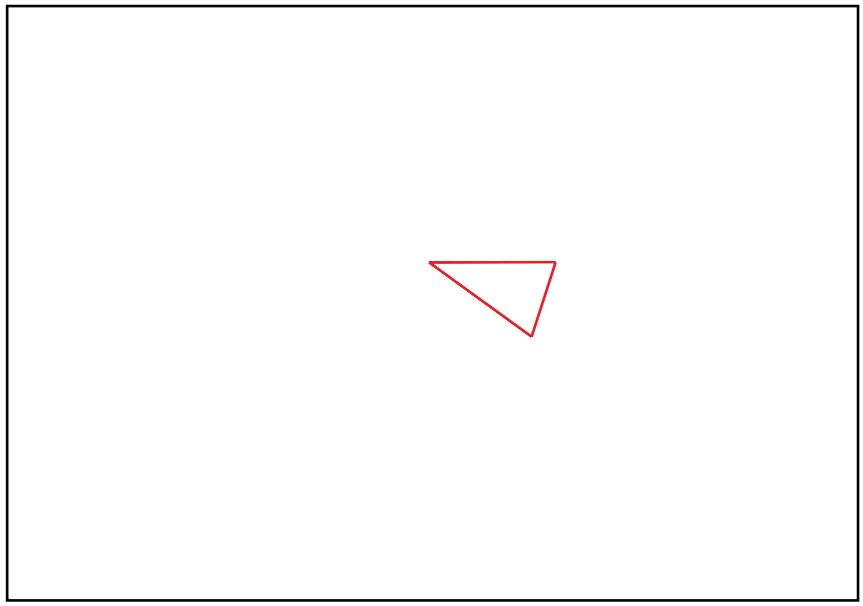
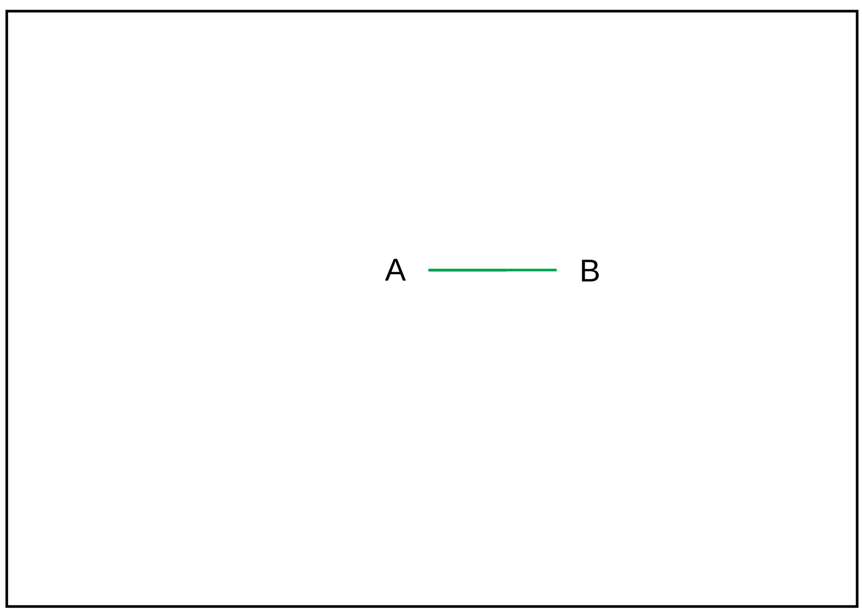
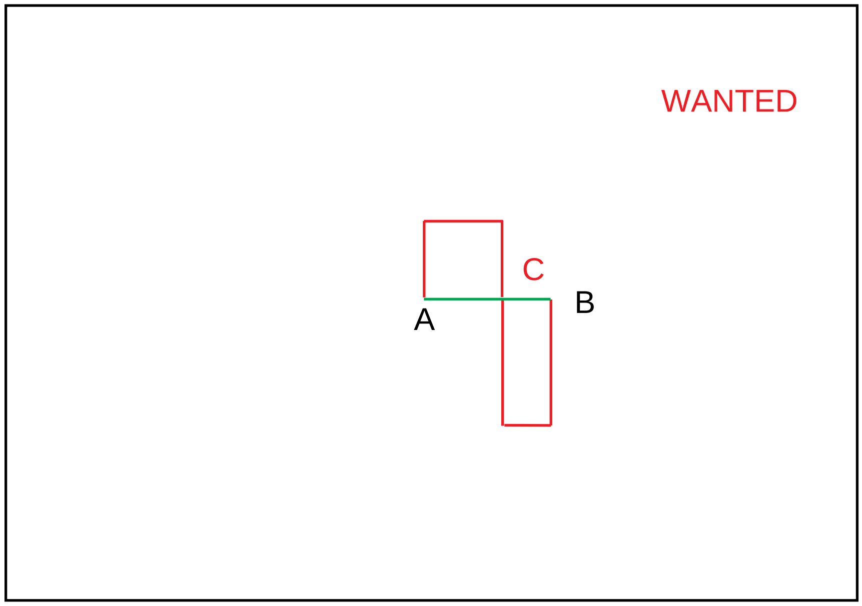
To construct an isoceles triangle having each of the angles at the base double of the remaining one.



IV.10:3. Let any straight line AB be set out,

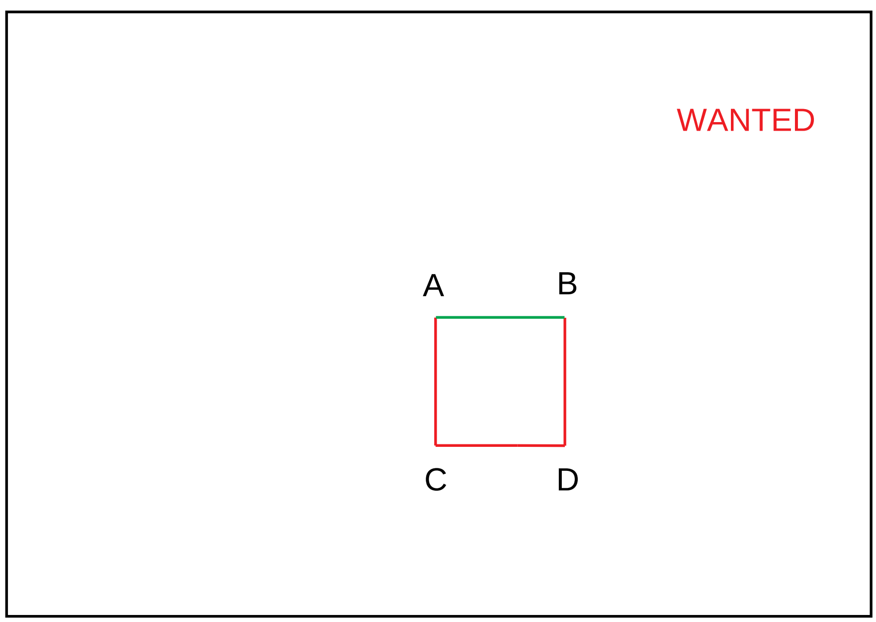


IV.10:3. and let it be cut at the point C so that the rectangle contained by AB, BC is equal to the square on CA; [II.11]



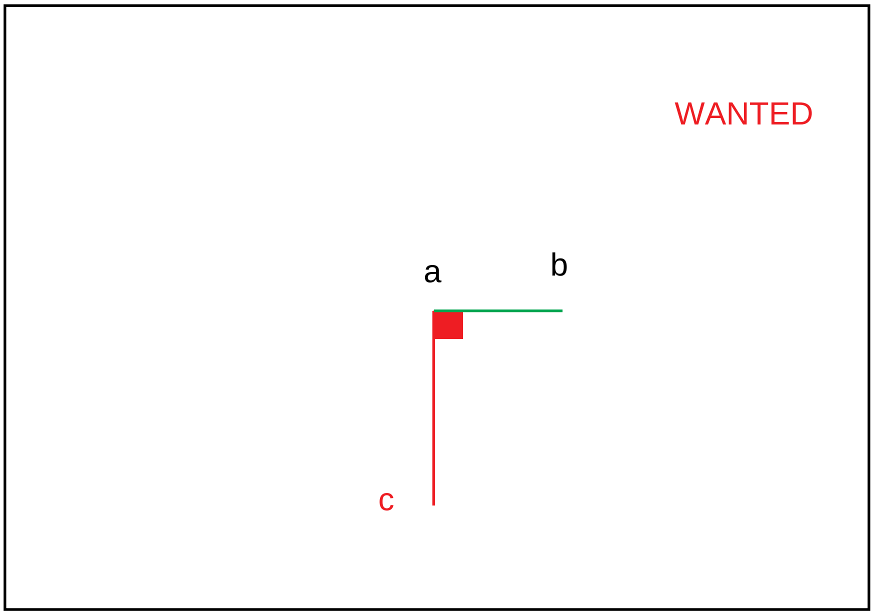
GOSUB II.11

II.11:9. For let the square ABCD be described on AB; [I.46].



GOSUB I.46. Relabel.

I.46:5. Let ac be drawn at right angles to the straight line ab from the point a on it [I.11],

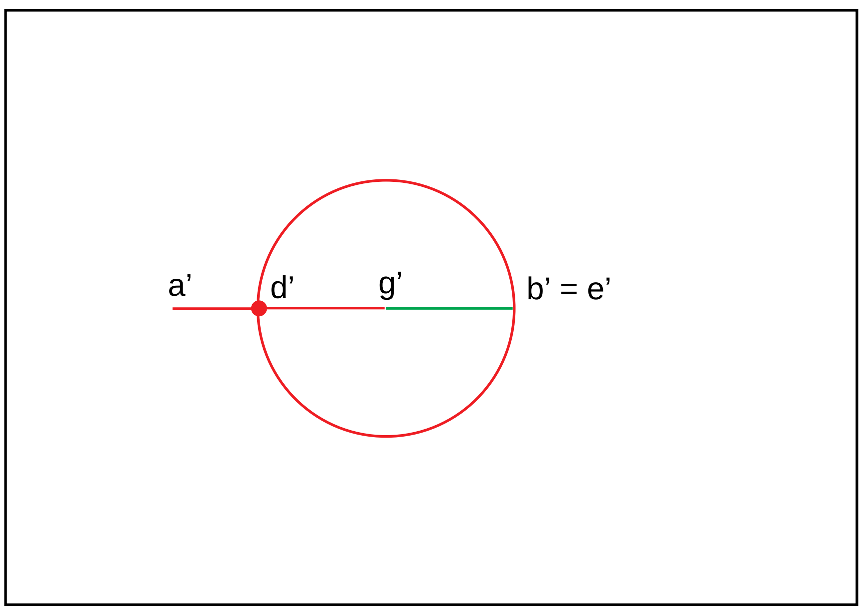


GOSUB I.11.
Extend the line.
Relabel.

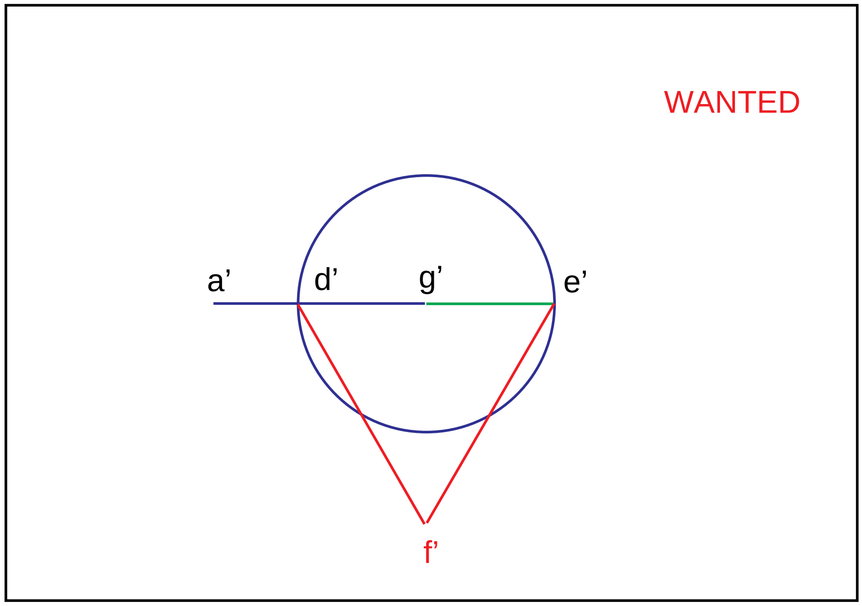
I.11:8. Let a point d’ be taken at random on a’g’;
let g’e’ be made equal to g’d’; [I.3]

NOTE. Rather than use dividers in place of I.3 as usual (cf. I.3) it will be useful here to use the compass, and draw a circle as shown.

Choose e’ = b’.
With centre g’ and distance g’e’ let the circle be described. Let d’ be the point through which this circle crosses a’g’.

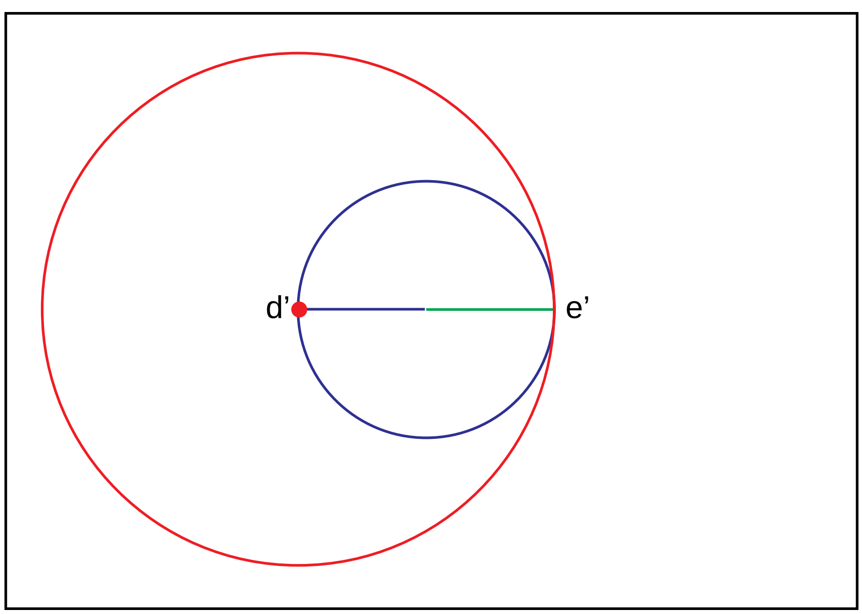


I.11:11. on d’e’ let the equilateral triangle f’d’e’ be constructed, [I.1]

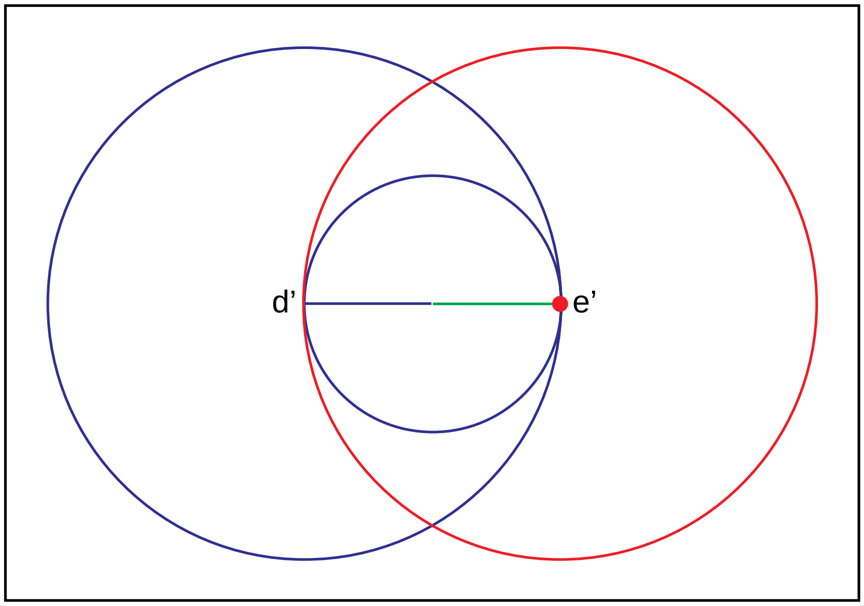


GOSUB I.1. We will relabel within I.1.

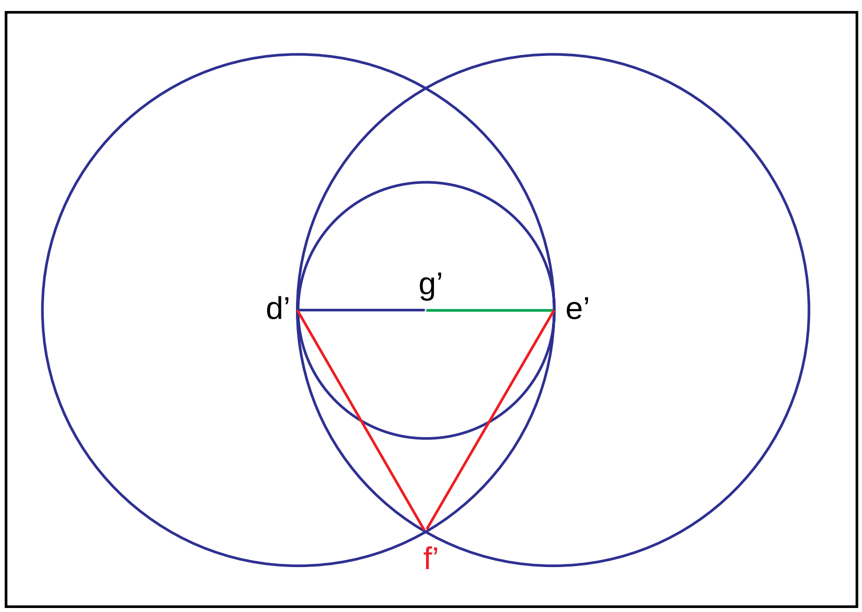
I.1:7. With centre d’ and distance d’e’ let the circle be described; [Post. 3]



I.1:10. again, with centre e’ and distance le’d’ et the circle be described; [Post. 3]



I.1:13. and from the point f’, in which the circles cut one another, [we choose the lower one] to the points d’, e’ let the straight lines f’d’, f’e’ be joined. [Post. 1]

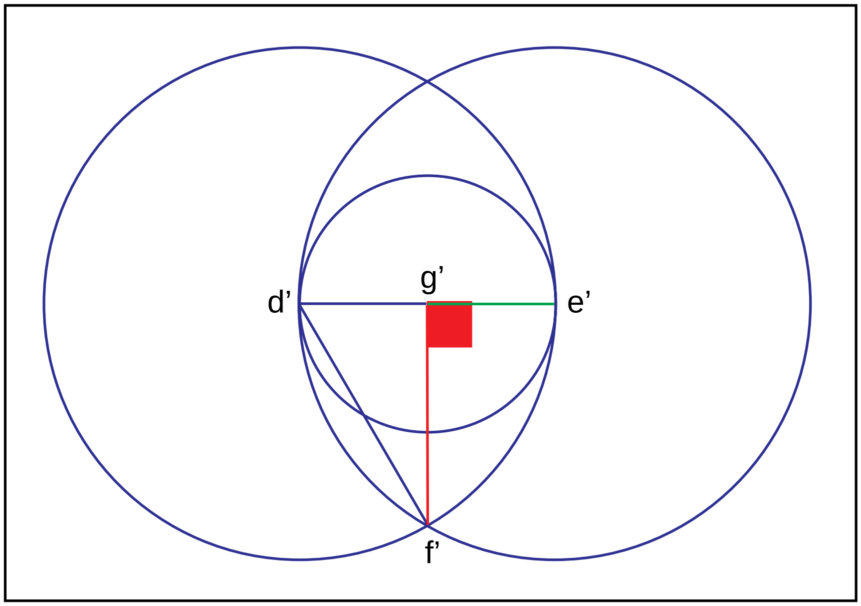


Cleanup. Retain the circle of step 1.
RETURN to I.11 at line 11.

I.11:13. and let f'g' be joined;

I say that the straight line has been drawn at right angles to the given straight line a'b' from the given point g' on it.

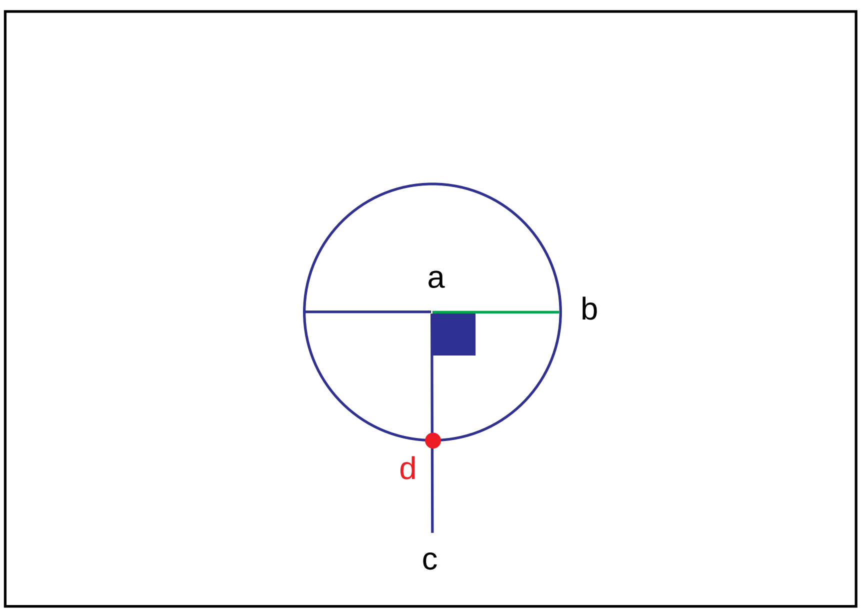
Cleanup. Retain the circle from step 1.
RETURN to I.46 at line 5.



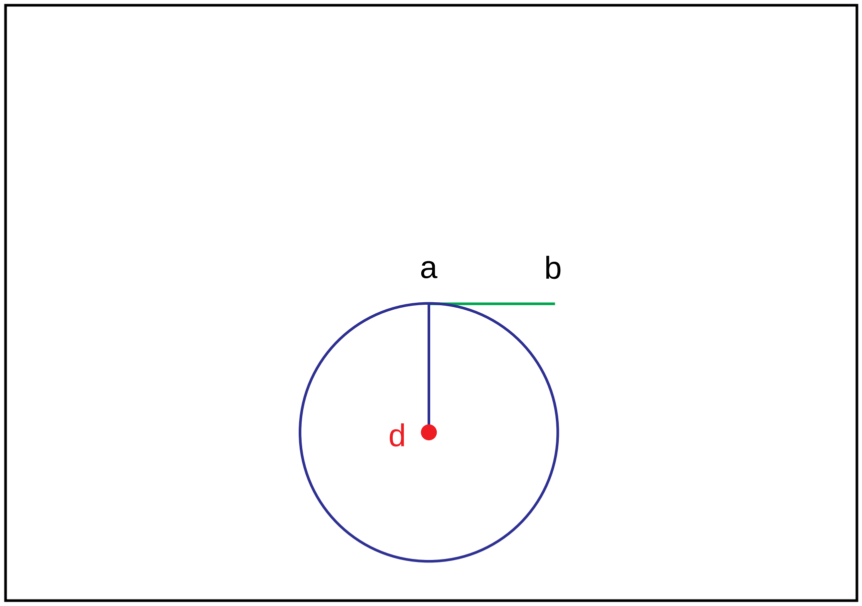
I.46:11. and let ad be made equal to ab;

This is where we make use of the circle retained from step 1.

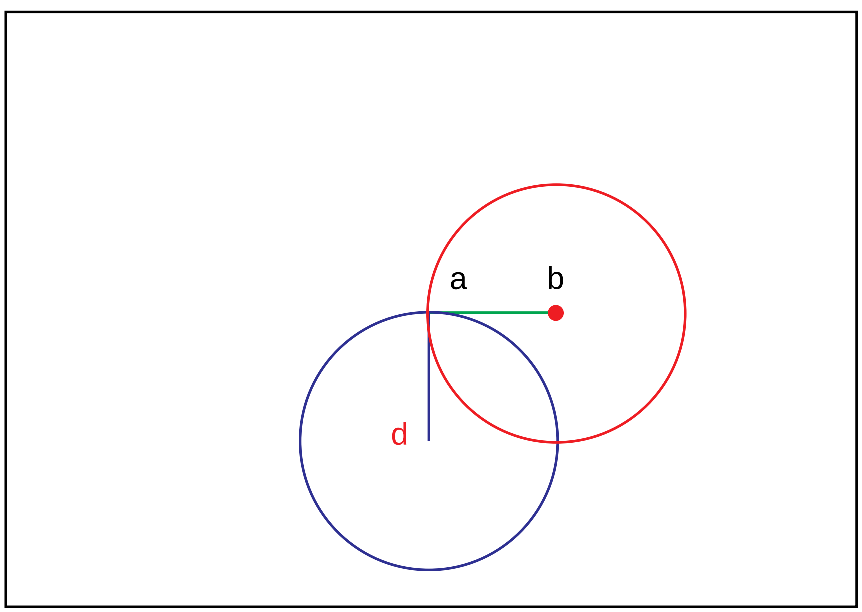
We may now save steps by following C#14B to complete the square.



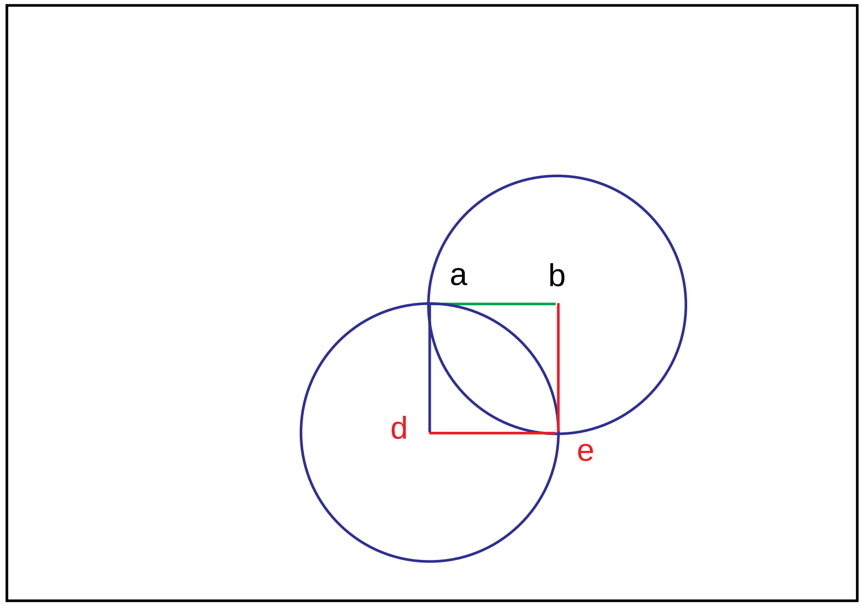
Swing da around d.



Swing ba around b.

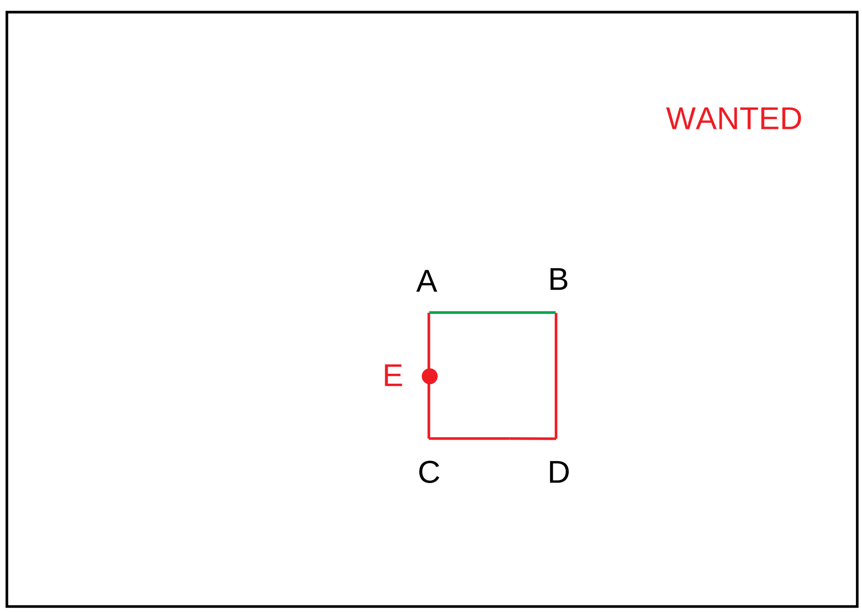


Connect the crossing point, e, to the centers.



Cleanup.
RETURN to I.11 at line 9.
Relabel.

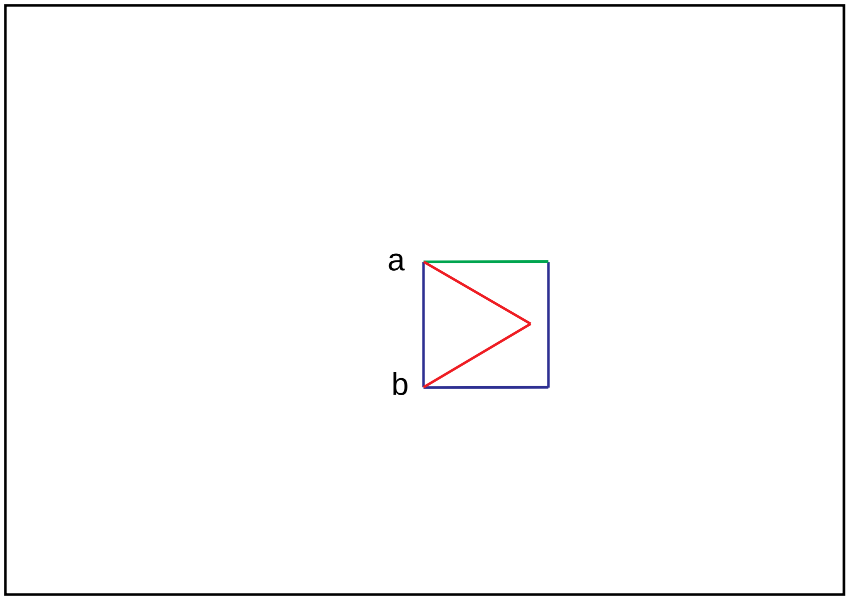
II.11:11. let AC be bisected at the point E, ([I.10])



GOSUB I.10.
Relabel.

I.10:2. Let ab be the given finite straight line.
I.10:4. Let the equilateral triangle abc be constructed on it, [I.1]
(For economy, we will follow the shortcut, C#5B.)

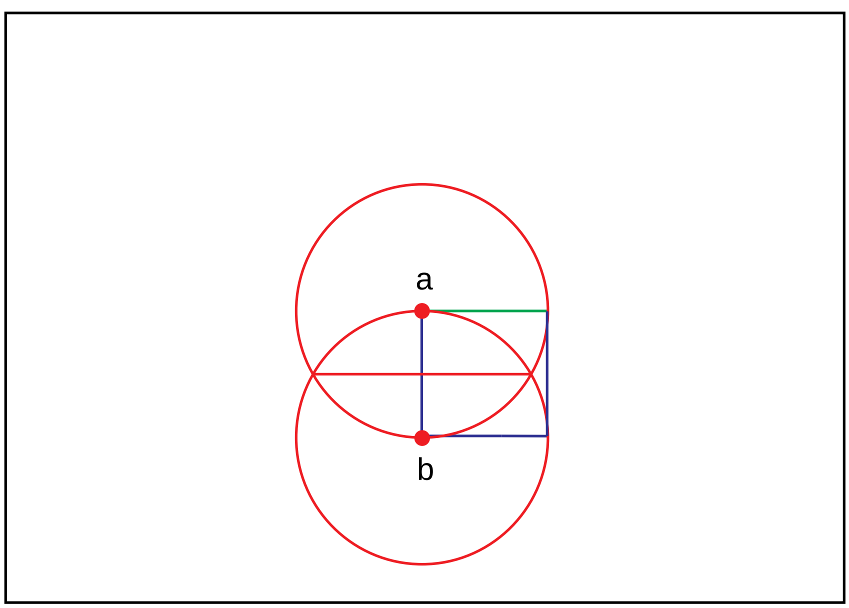
GOSUB I.1. (Vesica Pisces)



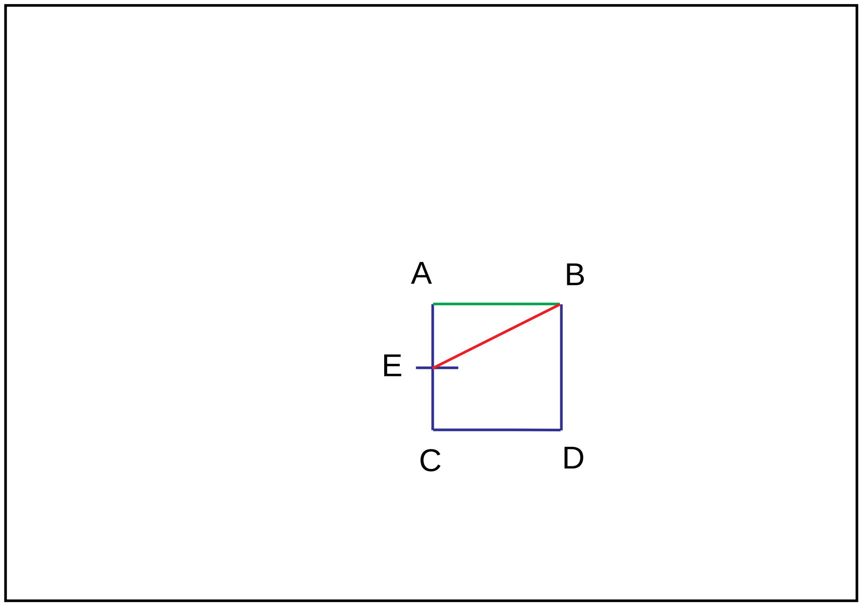
I.1:7. With centre a and distance ab let the circle be described;
(Revive step 6))

I.1:10. again, with centre b and distance ba let the circle be described;
(Revive step 9))
Connect the crossing points.
Mark the point where this line crosses ab.

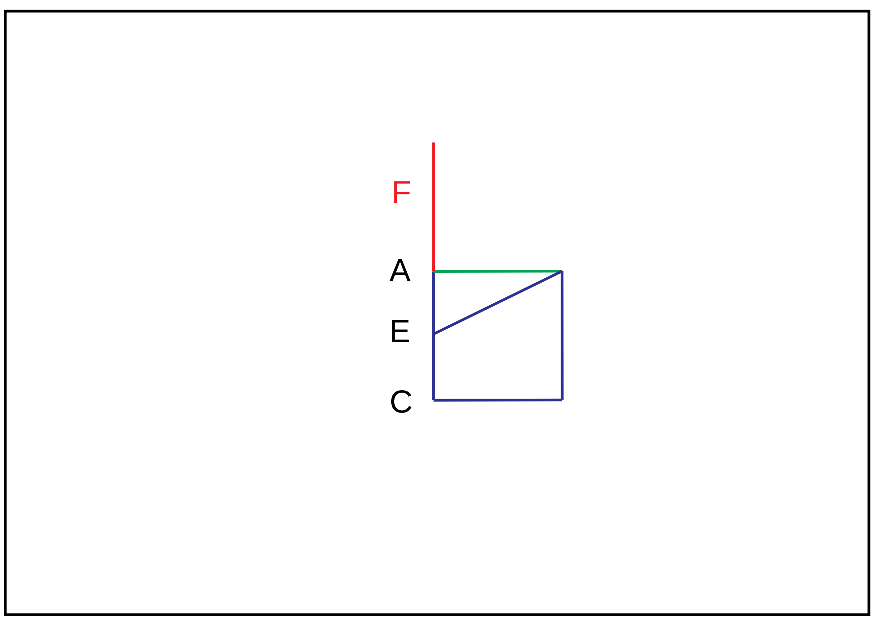
Cleanup and RETURN to I.11 at line 11.



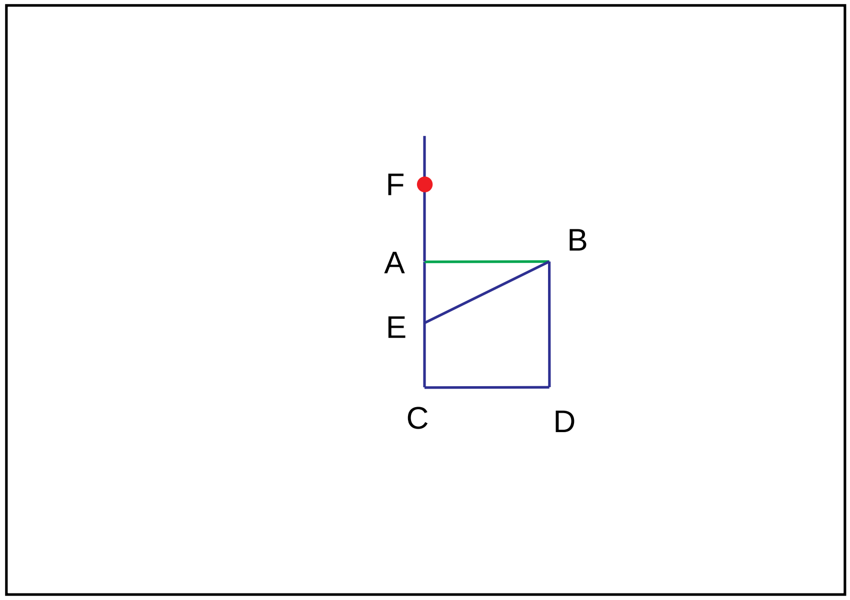
I.11:11. and let BE be joined;



I.11:12. let CA be drawn through to F, (this point is not actually located.)

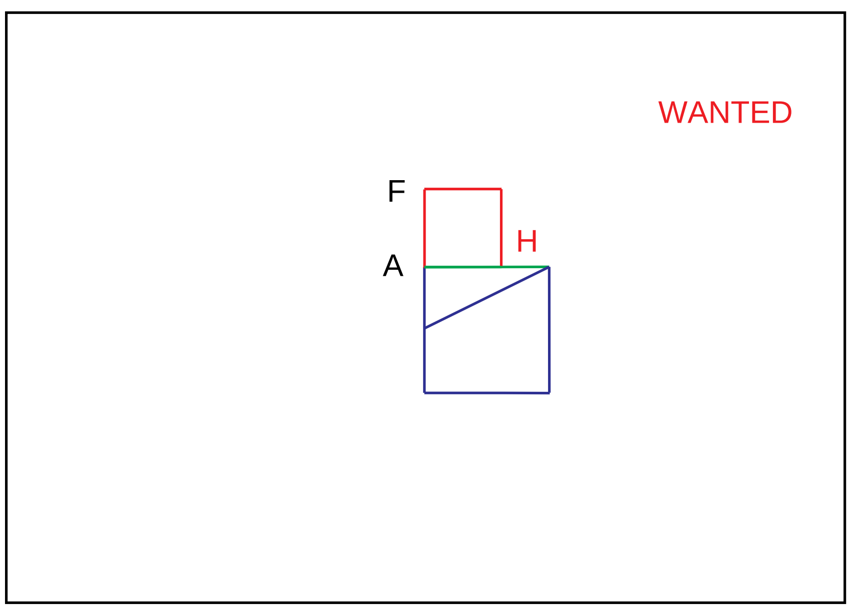


II.11:12. and let EF be made equal to BE; ([I.3], or dividers)

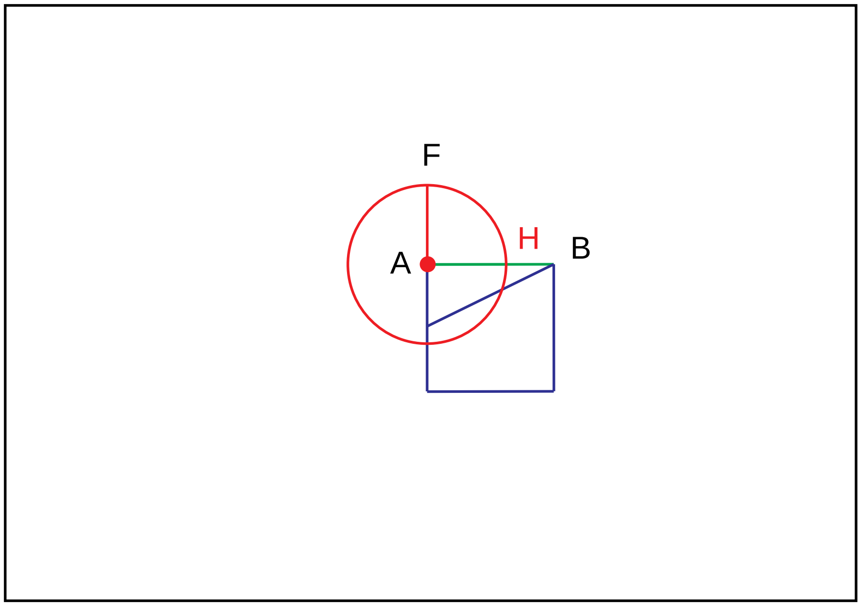


II.11:14. let the square FH be described on AF,

GOSUB I.46. (We will use the variation C#14B: swing three arcs with the same compass.)



Swing from the corner, A.
Locate the point H on AB so AH is equal to AF. (or, dividers.)

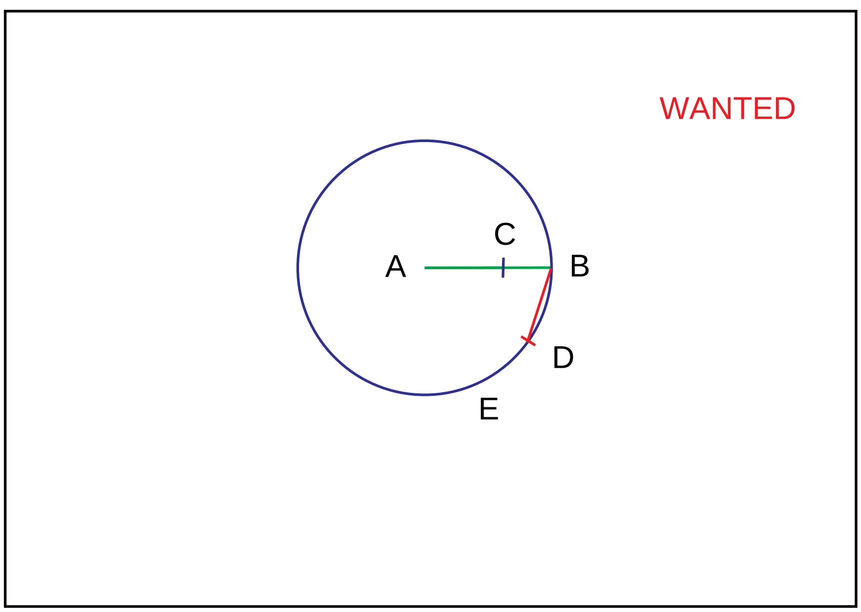


NOTE. With this point we are finished with the first instruction: To cut a given straight line. This is the golden section in 14 steps. We include the next steps, formally part of the machinery of the proof, to see what we have done.

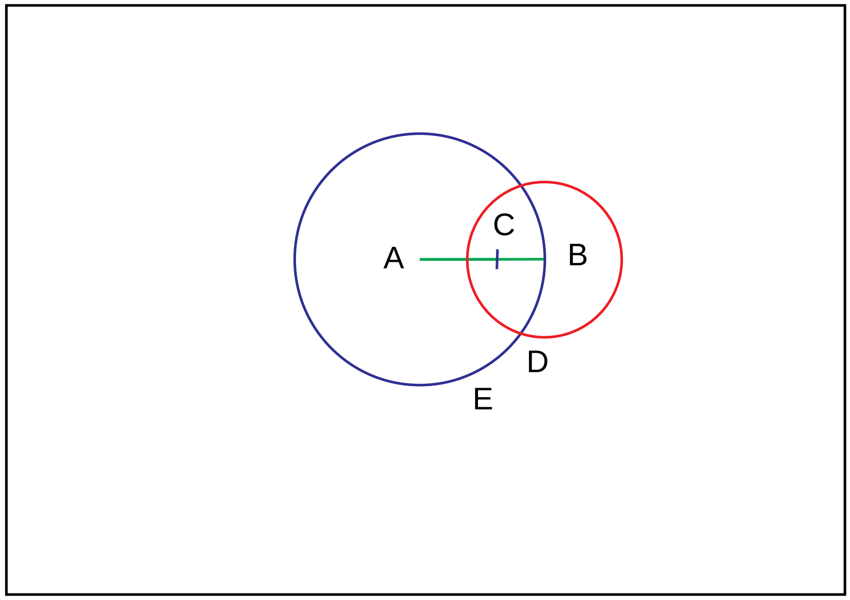
Cleanup.
RETURN to IV.10:3.
Relabel.

IV.10:7. with centre A and distance AB let the circle BDE be described,
(Recall this circle from step 1).

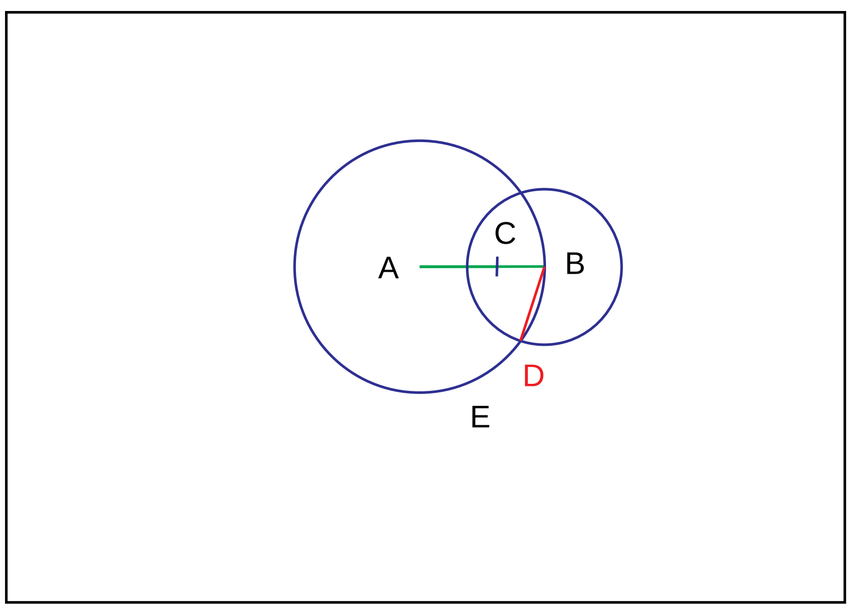
IV.10:9. and let there be fitted in the circle BDE the streaight line BD equal to the straight line AC which is not greater than the diameter of the circle BDE. [IV.1]



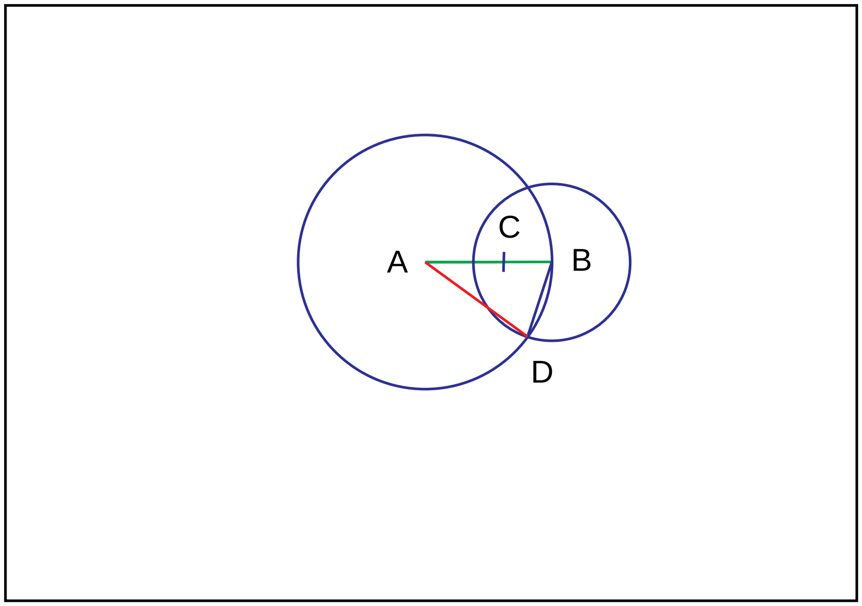
Set the compass to distance AC and draw the circle with centre B and distance AC.



Let D be the lower point at which the circles cross. Let BD be joined.

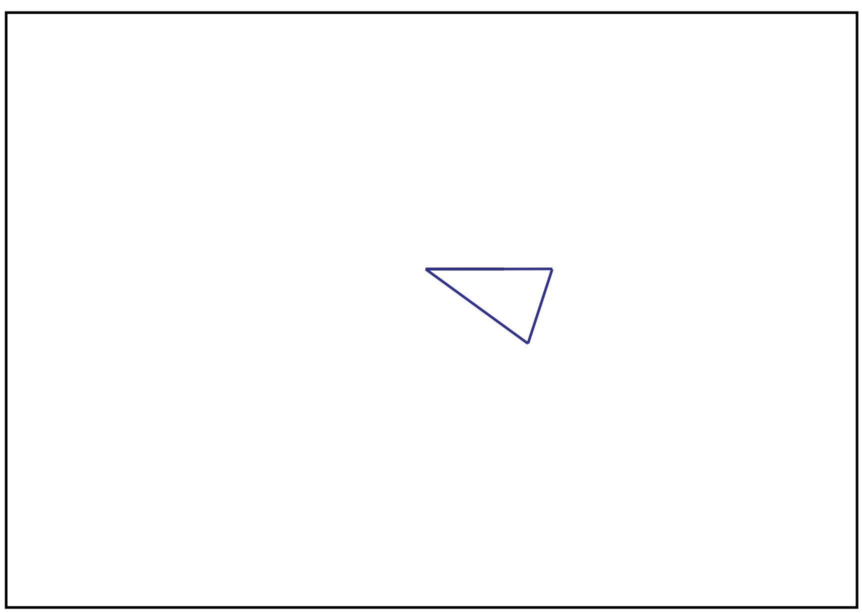


Cleanup.
RETURN to IV.10:9.



IV.10:14. Let AD, DC be joined,

We need only AD, and we are done.



Q.E.F.