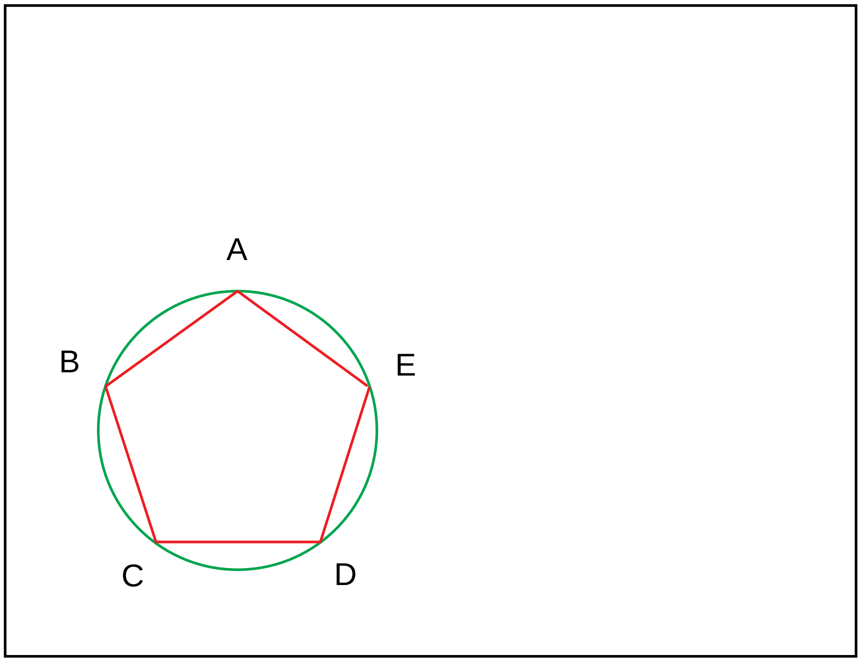
Construction 33: Book IV, Proposition 11

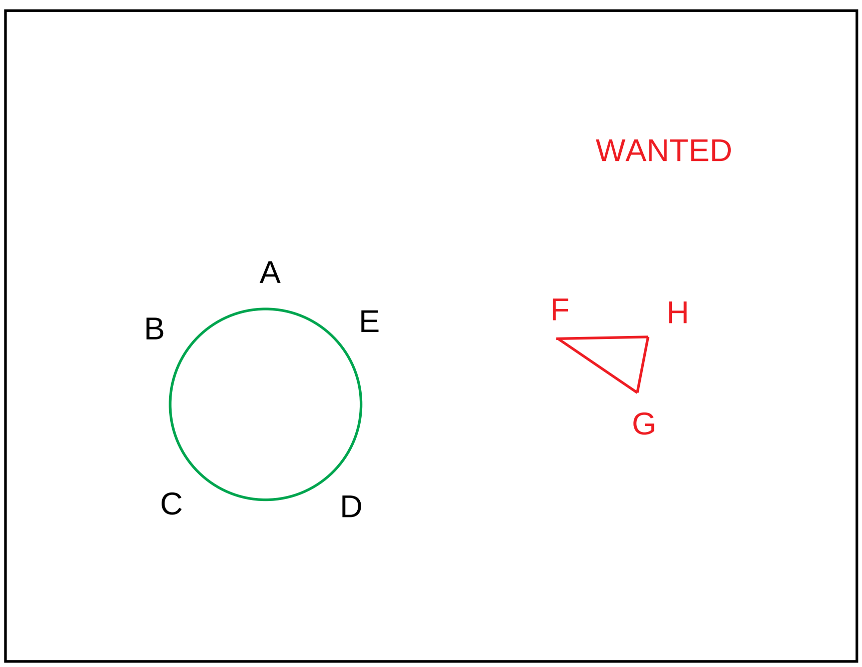
In a given circle to inscribe an equilateral and equiangular pentagon.



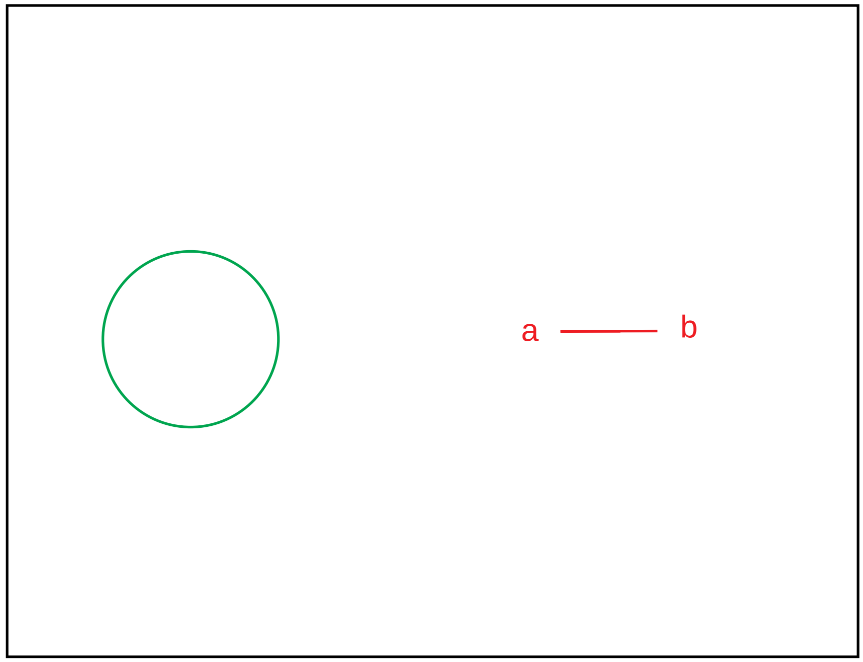
IV.11:3. Let ABCDE be the given circle;

IV.11:6. Let the isoceles triangle FGH be set out having each of the angles at G, H double of the angle at F; [IV.10]

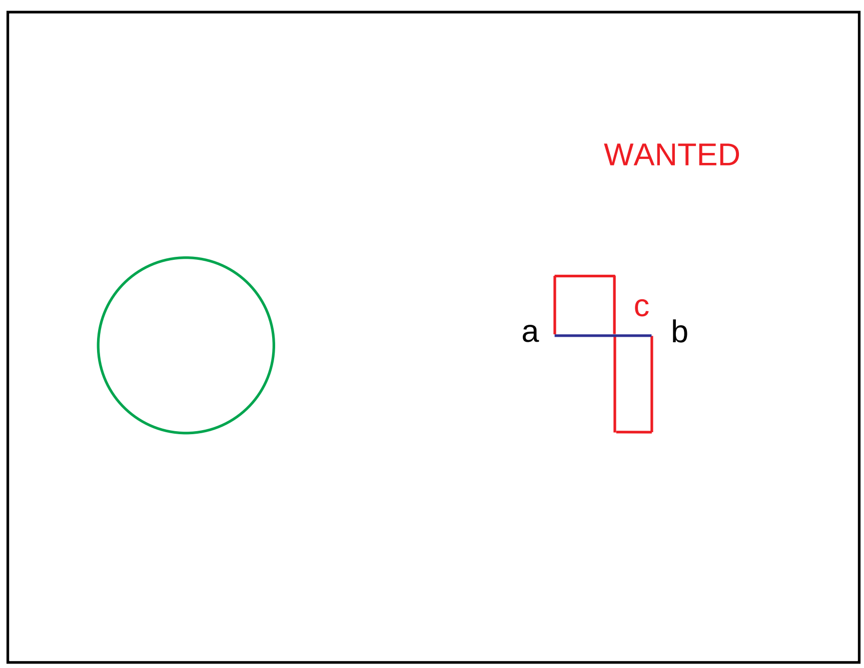
GOSUB IV.10.
Relabel.



IV.10:3. Let any straight line ab be set out,

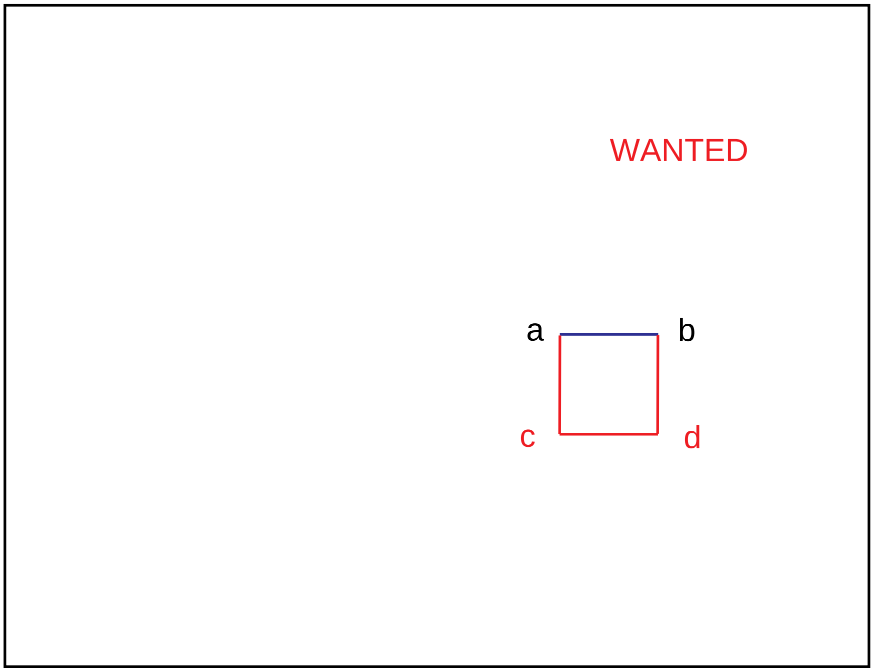


IV.10:3. and let it be cut at the point C so that the rectangle contained by AB, BC is equal to the square on CA; [II.11]



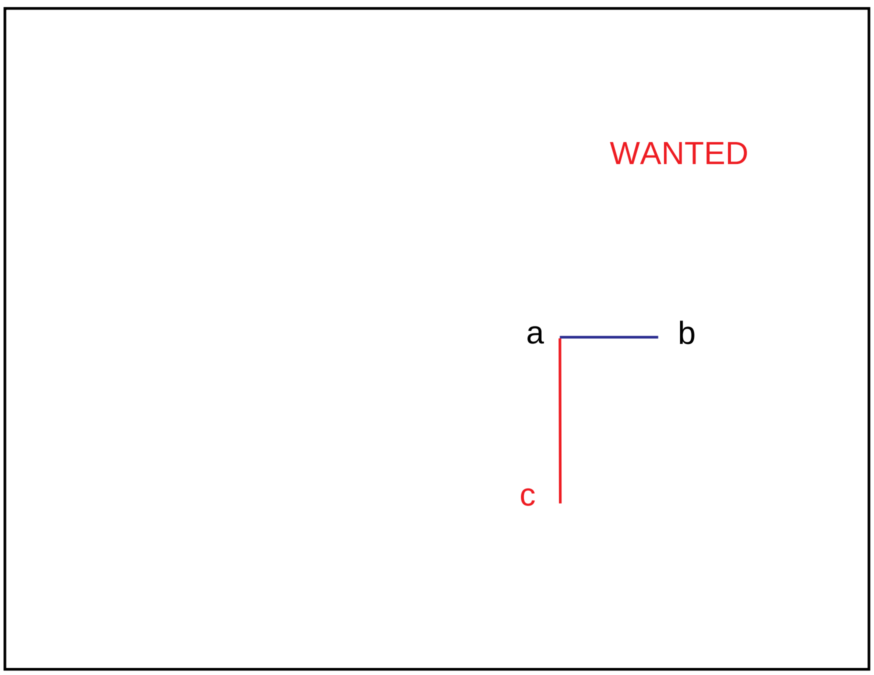
GOSUB I.11.
Extend the line.
Relabel.

II.11:9. For let the square ABCD be described on AB; [I.46].



GOSUB I.46. Relabel.

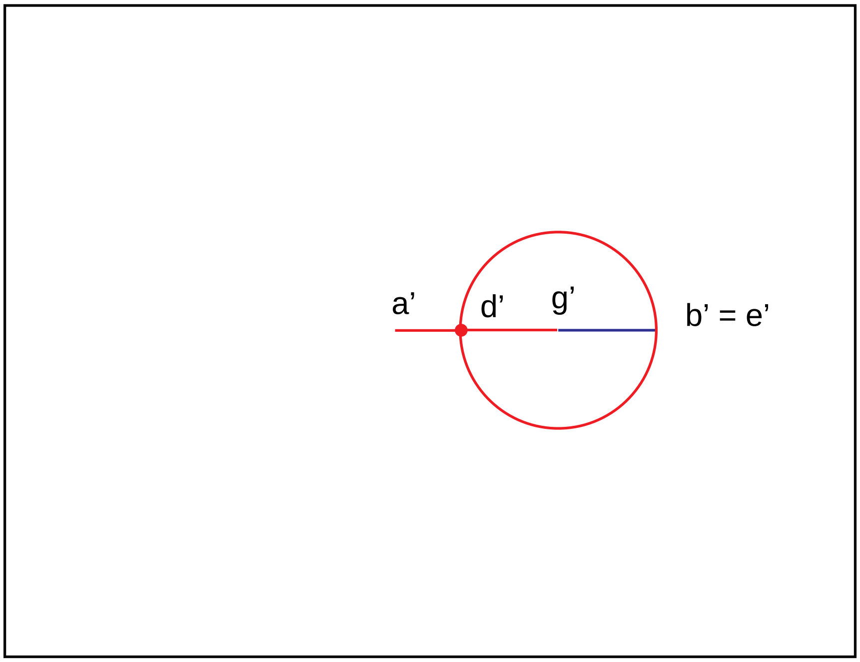
I.46:5. Let ac be drawn at right angles to the straight line ab from the point a on it [I.11],



GOSUB I.11.
Extend the line.
Relabel.

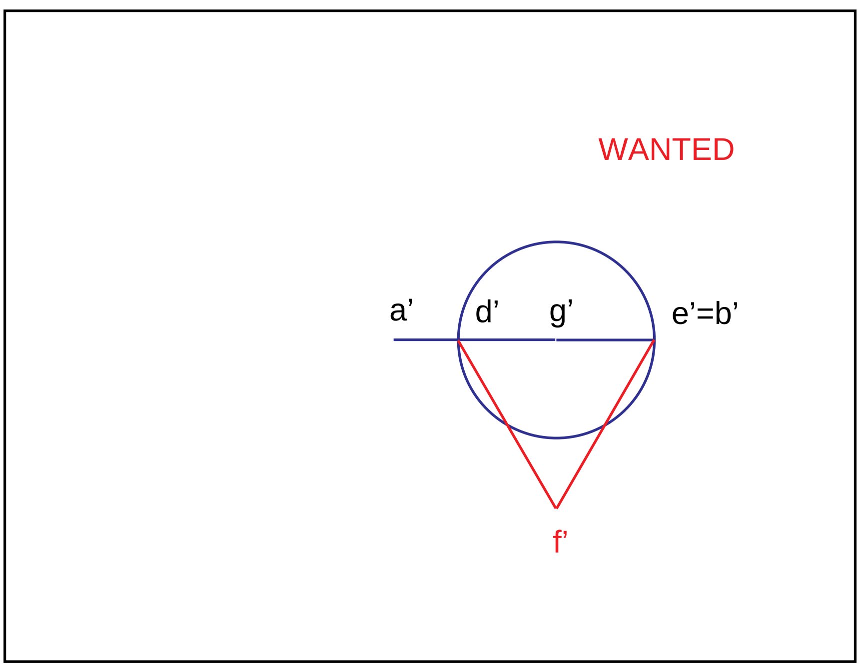
I.11:8. Let a point d’ be taken at random on a’g’;
let d’g’ be made equal to g’e’; [I.3]

NOTE. Rather than use dividers in place of I.3 as usual (cf. I.3) it will be useful here to use the compass, and draw a circle as shown.
Choose e’ = b’.
With centre g’ and distance g’e’ let the circle be described. Let d’ be the point through which this circle crosses a’g’.

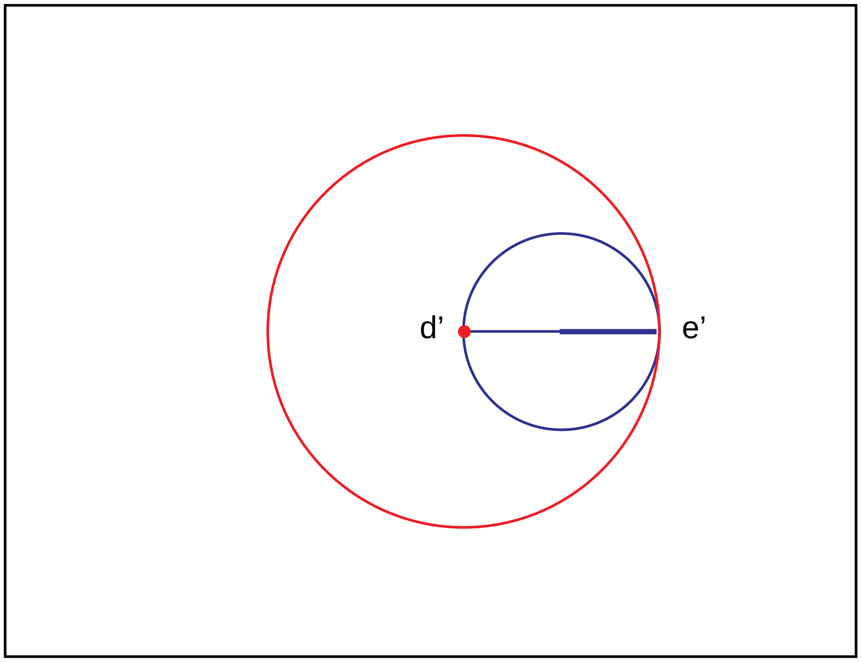


I.11:11. on d’e’ let the equilateral triangle f’d’e’ be constructed, [I.1]

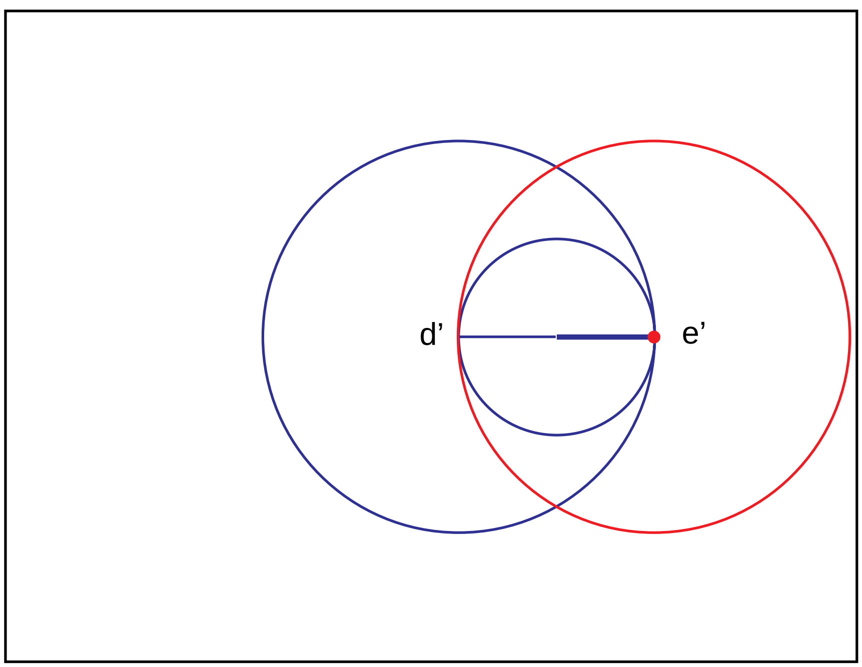
GOSUB I.1. We will relabel within I.1.



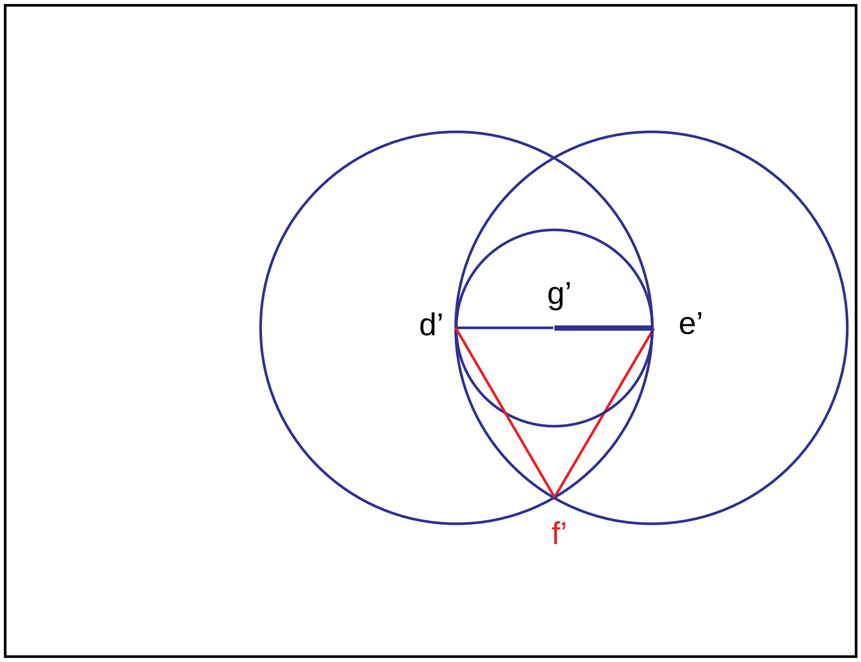
I.1:7. With centre d’ and distance d’e’ let the circle be described; [Post. 3]



I.1:10. again, with centre e’ and distance e’d’ et the circle be described; [Post. 3]



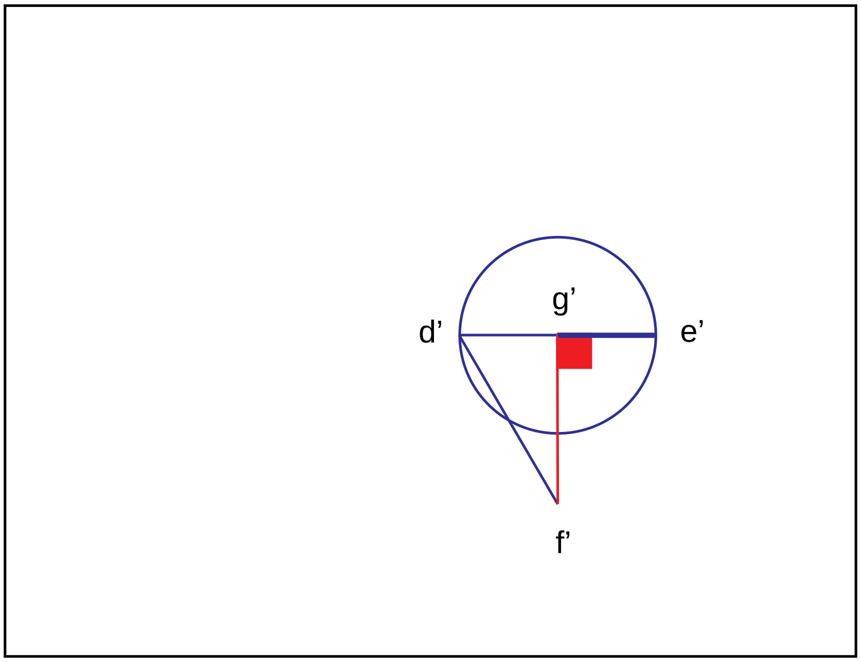
I.1:13. and from the point f', in which the circles cut one another, [we choose the lower one] to the points d', e' let the straight lines f'd', f'e' be joined. [Post. 1]



Cleanup. Retain the circle from step 1.

RETURN to I.11 at line 11.

I.11:13. and let f'g' be joined;
I say that the straight line f'g' has been drawn at right angles to the given straight line a'b' from g' the given point on it.

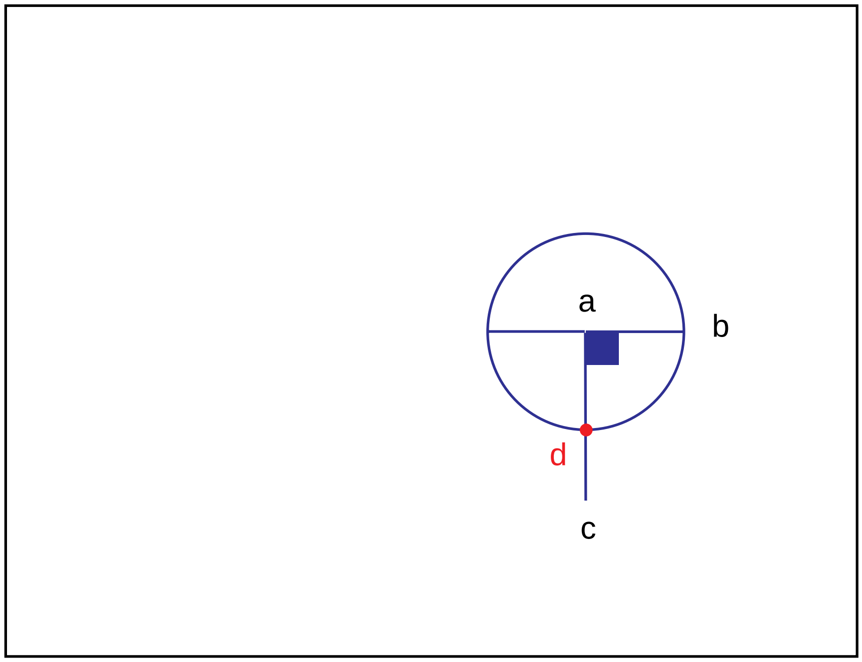


Cleanup. Retain the circle from step 1.

RETURN to I.46 at line 5.

I.46:11. and let ad be made equal to ab;

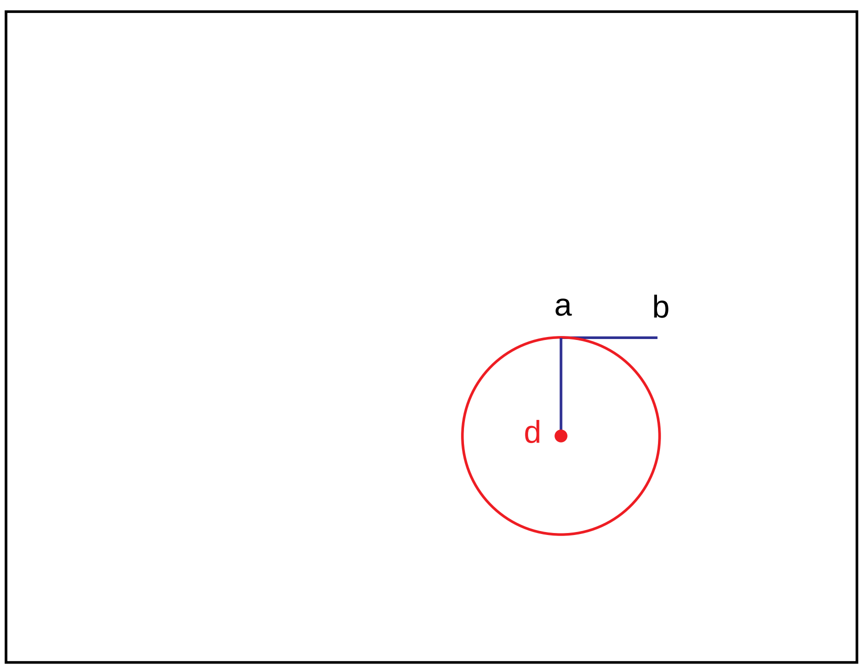
This is where we make use of the circle retained from step 1.



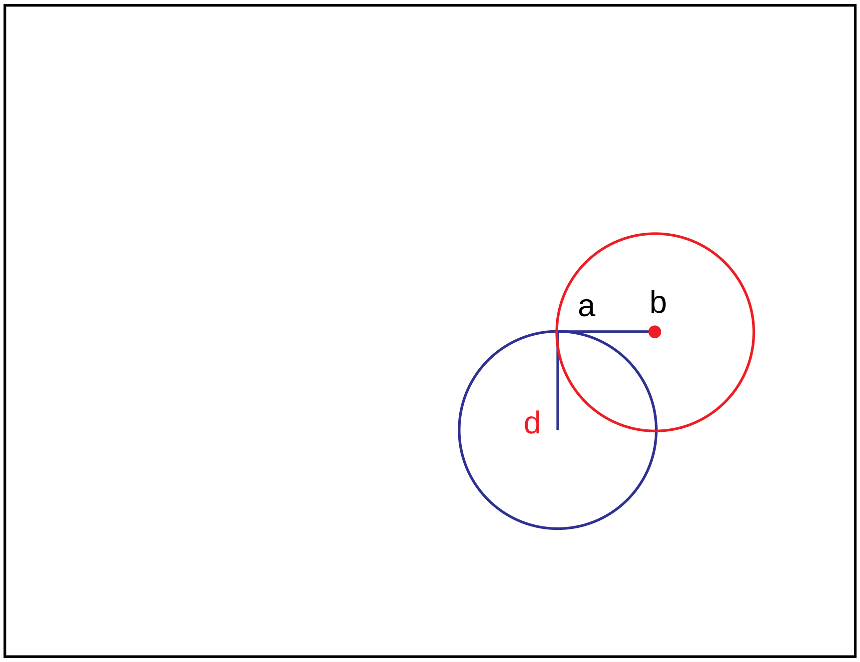
Cleanup.

We may now save steps by following C#14B to complete the square.

Swing da around d.

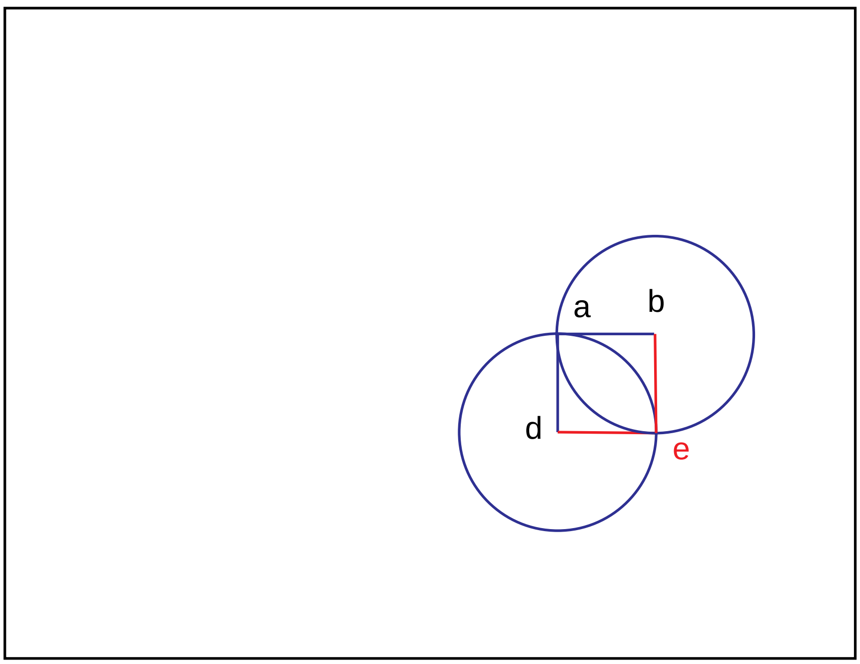


Swing ba around b.



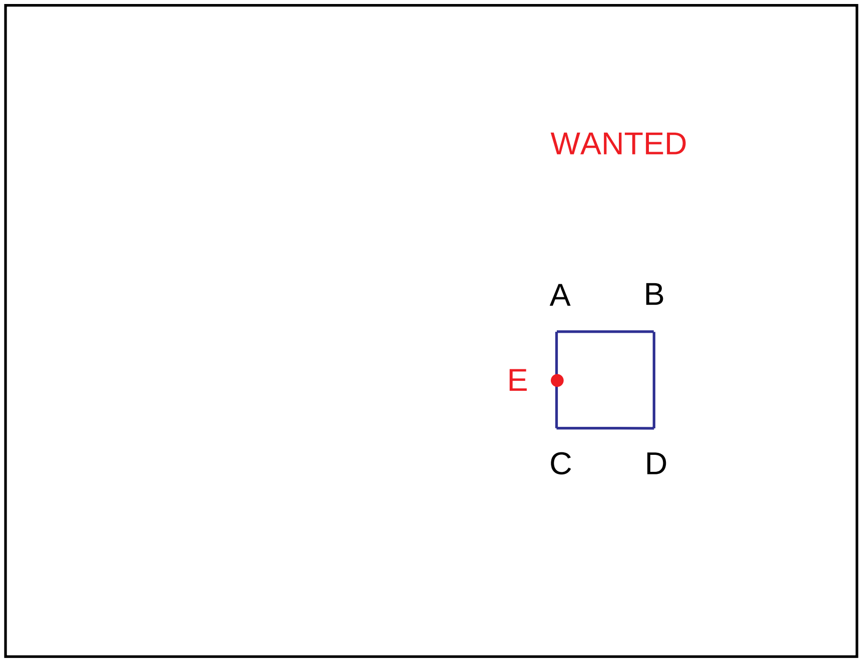
Connect the crossing point, e, to the centers.

Cleanup.
RETURN to I.11 at line 9.
Relabel.



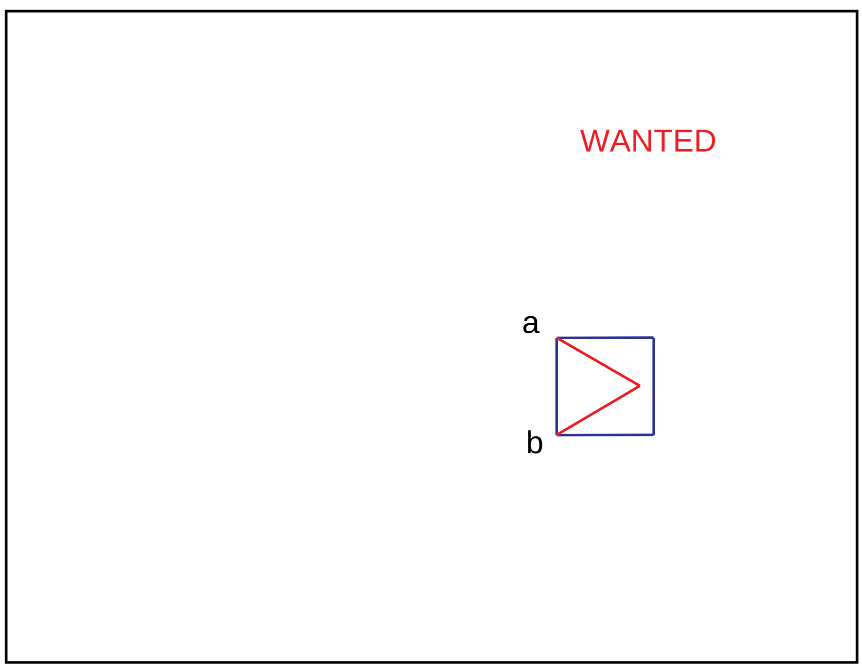
II.11:11. let AC be bisected at the point E, ([I.10])

GOSUB I.10.
Relabel.

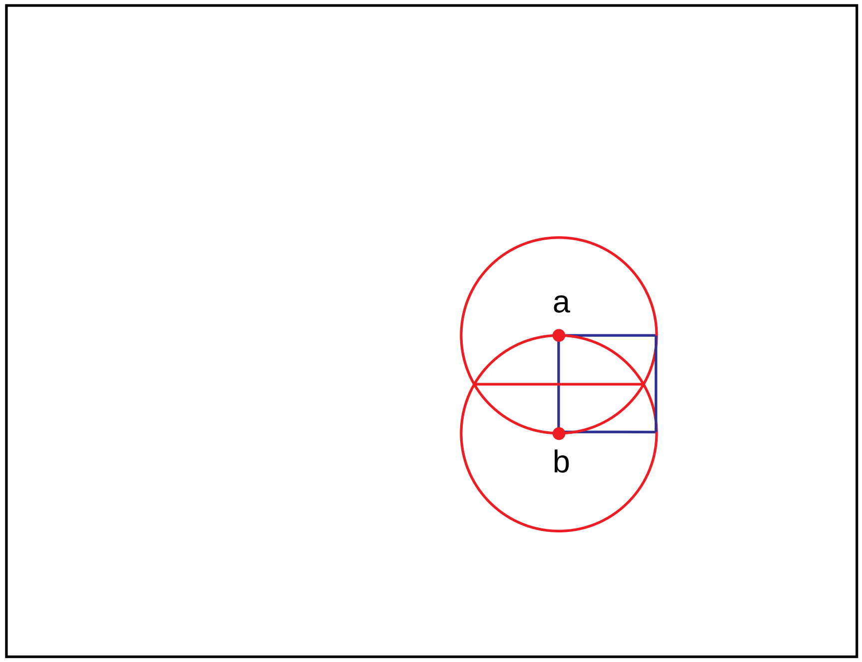


I.10:2.. Let ab be the given finite straight line.
I.10:4. Let the equilateral triangle abc be constructed on it, [I.1] (For economy, we will follow the shortcut, C#5B.)

GOSUB I.1 (Vesica Pisces)

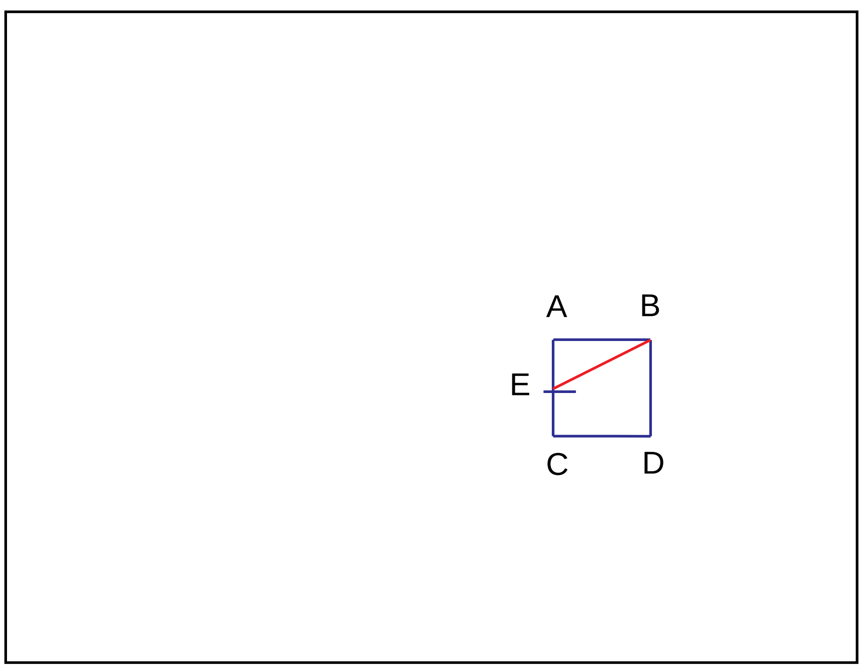


I.1:7. With centre a and distance ab let the circle be described;
(Revive step 6))

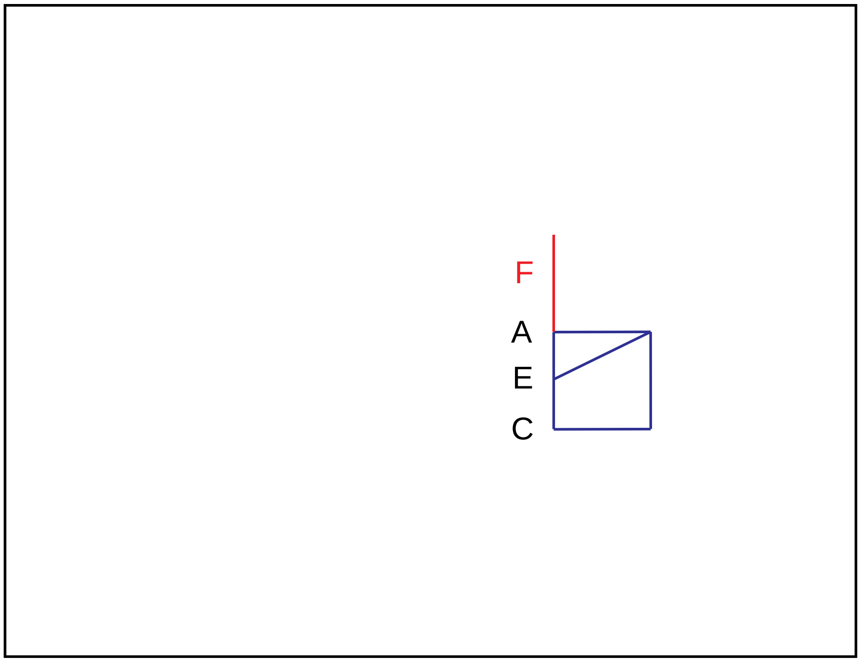


I.1:10. again, with centre b and distance bs let the circle be described;
(Revive step 9))
Connect the crossing points.
Mark the point where this line crosses ab.
Cleanup and RETURN to I.11 at line 11.

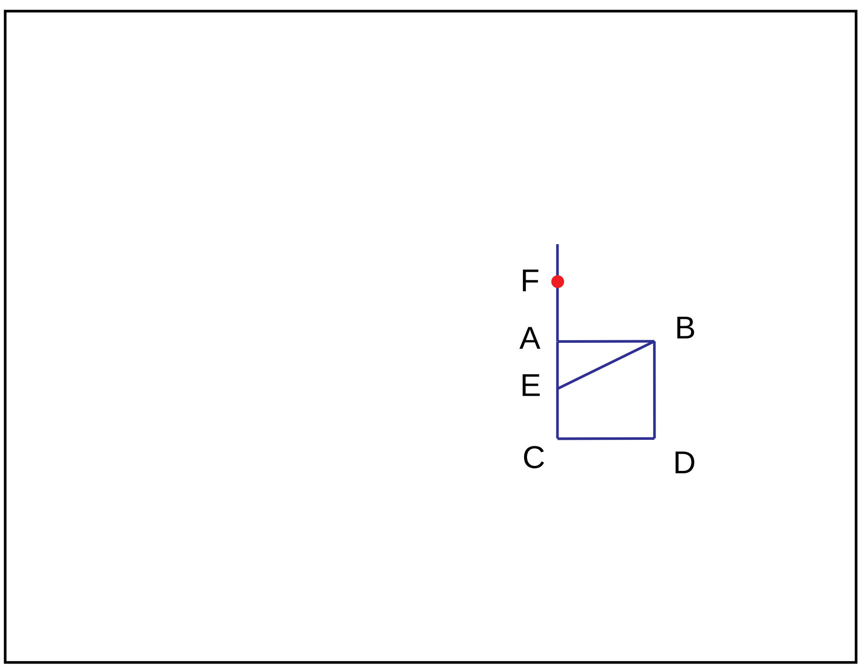
I.11:11. and let BE be joined,



II.11:12. let CA be drawn through to F, (this point is not actually located.)

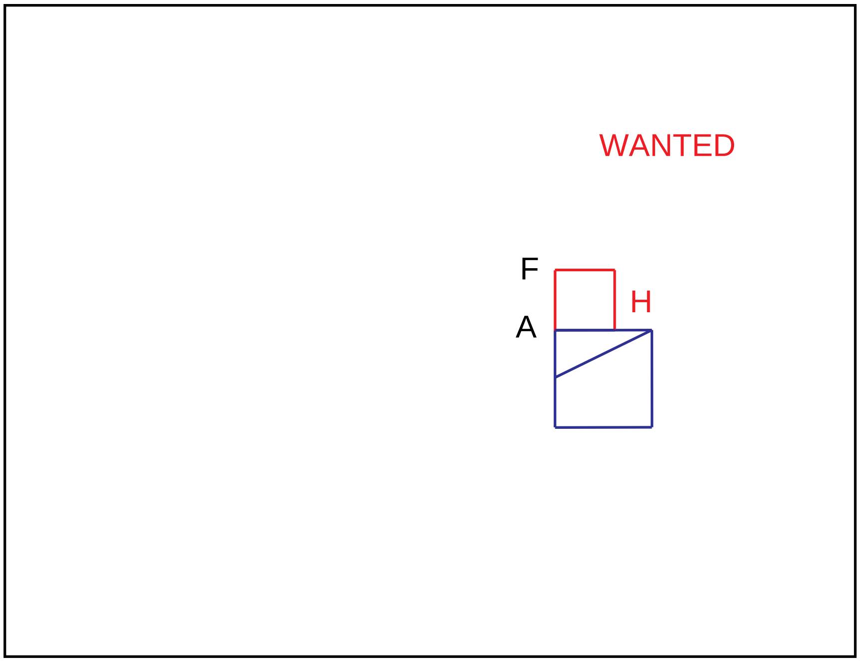


II.11:12. and let EF be made equal to BE; (I.3], or dividers)



II.11:14. let the square FH be described on AF,

GOSUB I.46 (We will use the variation C#14B: swing three arcs with the same compass.)

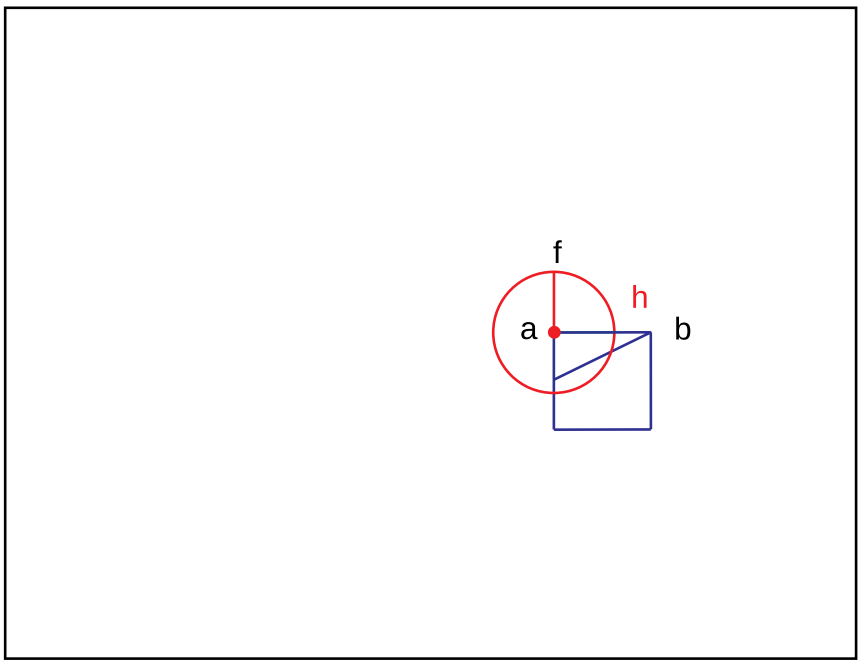


Swing from the corner, a. Locate the point h on ab so ah is equal to af. (or, dividers.)

NOTE. With this point we are finished with the first instruction: To cut a given straight line. This is the golden section in 14 steps.

Cleanup.

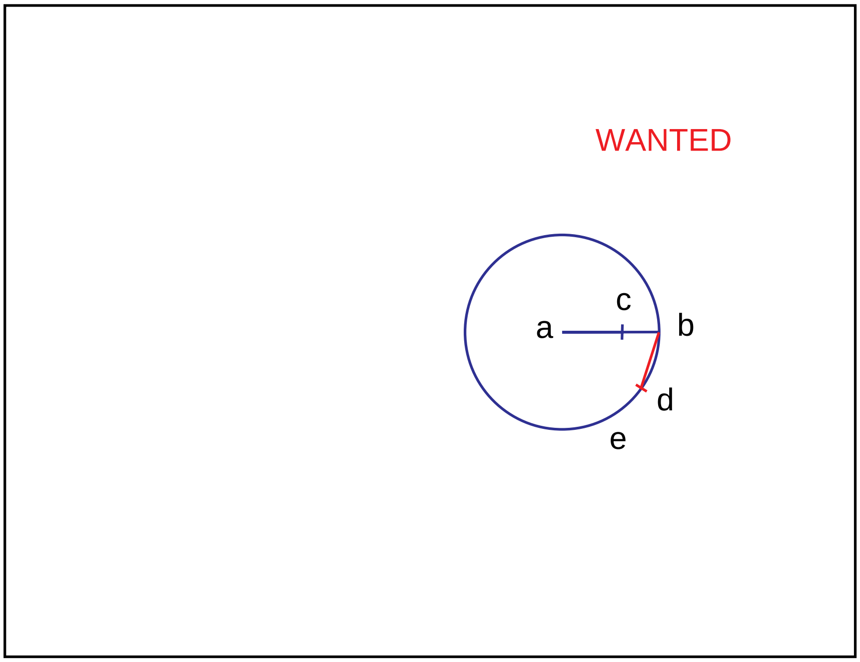
RETURN to IV.10:3.
Relabel.



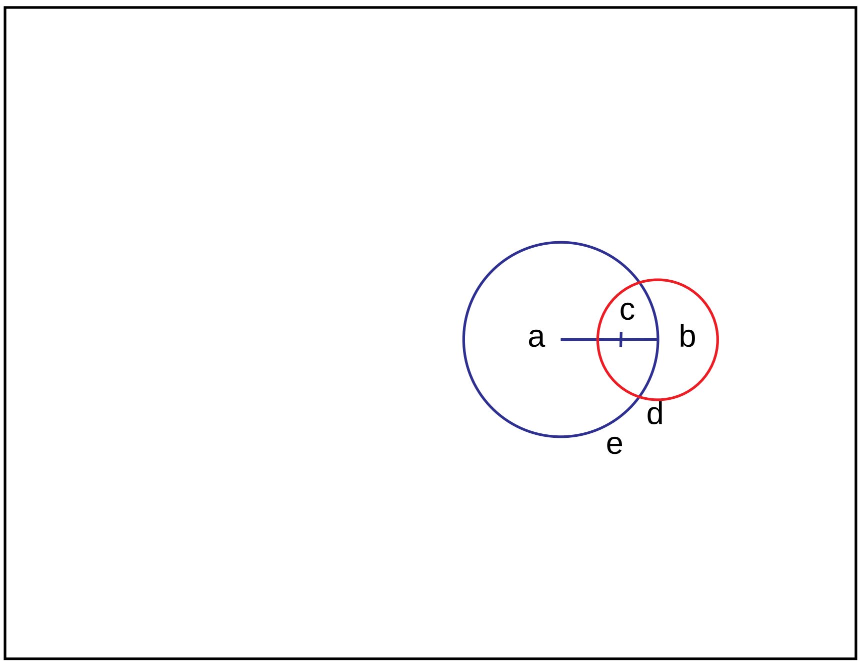
IV.10:7. With centre a and distance ab let the circle bde be described,

(Recall this circle from step 1.)

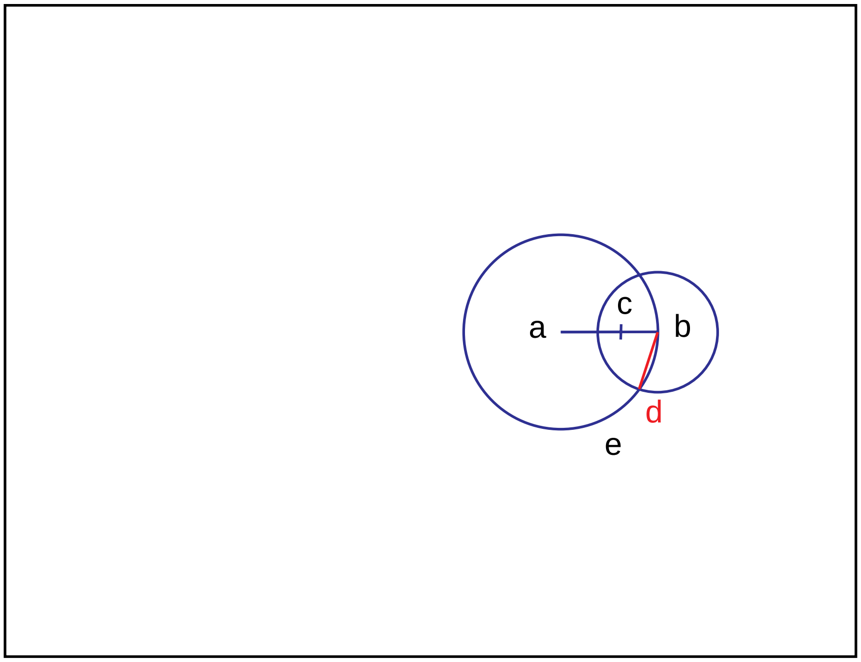
IV.10:9. and let there be fitted in the circle bde the straight line bd equal to the straight line ac which is not greater than the diameter of the circle bde. [IV.1]



Set the compass to distance ac and draw the circle with centre b and distance ac.

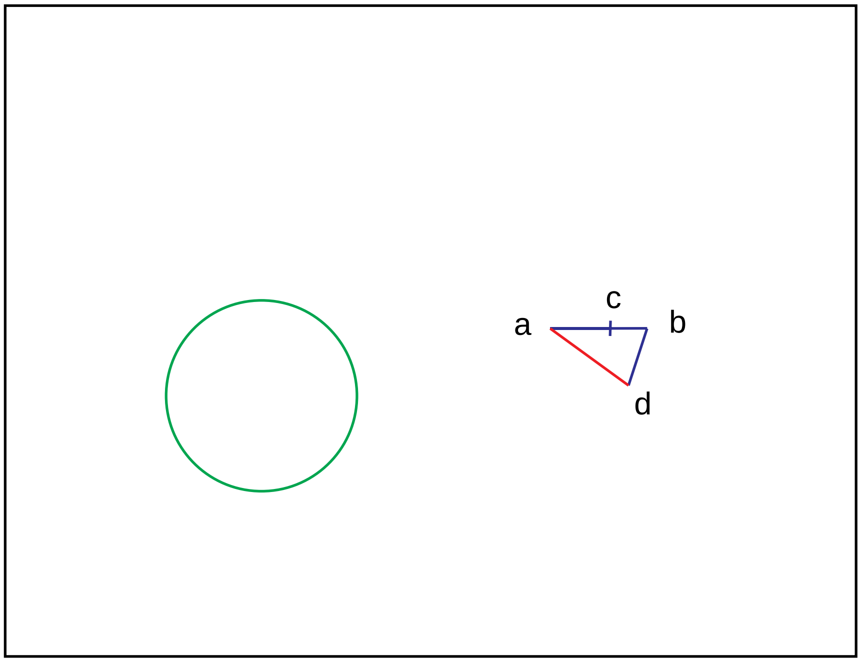


Let d be the lower point at which the circles cross. Let bd be joined.



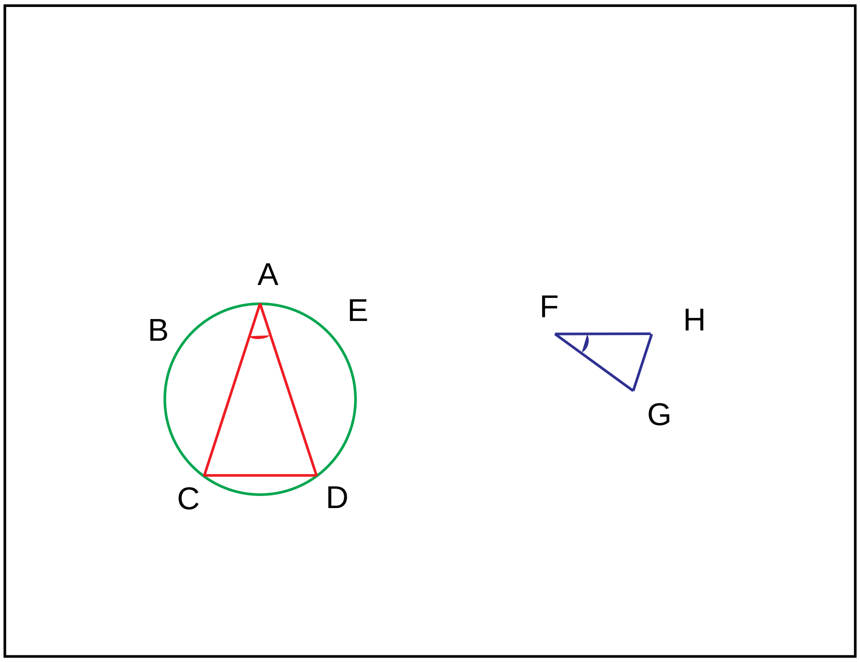
Cleanup.
RETURN to IV.10:9.

IV.10:14. Let ad be joined.



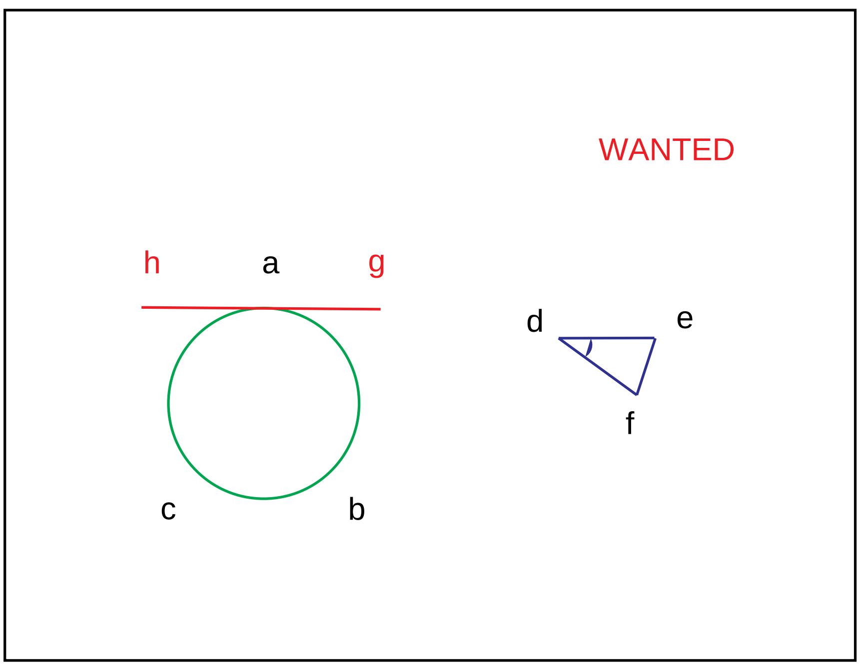
RETURN to IV.11 at line 6.
Relabel.

IV.11:10. let there be inscribed in the circle ABCDE the triangle ACD equiangular with the triangle FGH, so that the angle CAD is equal to the angle at F and the angles at G, H respectively equal to the angles ACD, CDA; [IV.2]



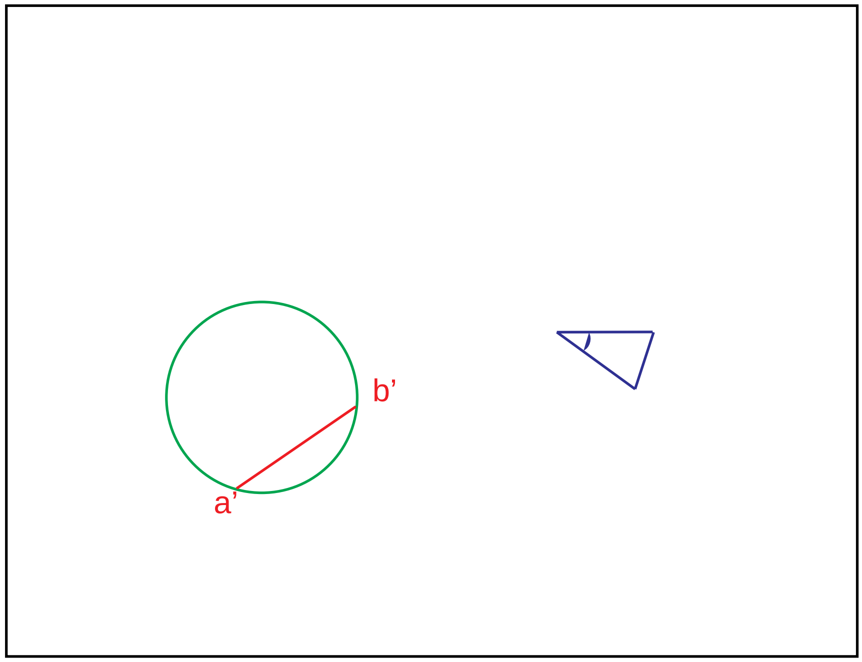
GOSUB IV.2
Relabel. We choose a at the top of the given circle, and d at the smaller angle of the golden triangle; d will be moved to a.

IV.2:7. Let gh be drawn touching the circle abc at a. [III.17] (C#18B)



GOSUB C#18B.

III.17:6. Let the centre e of the circle be taken, [III.1]

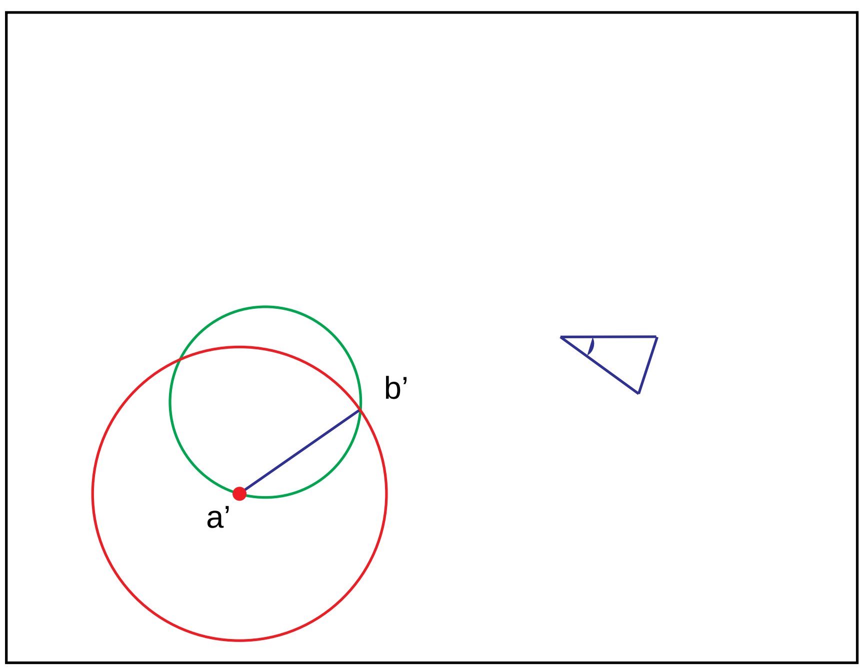


GOSUB III.1.
III.1:4. Let a straight line a’b’ be drawn through it at random,

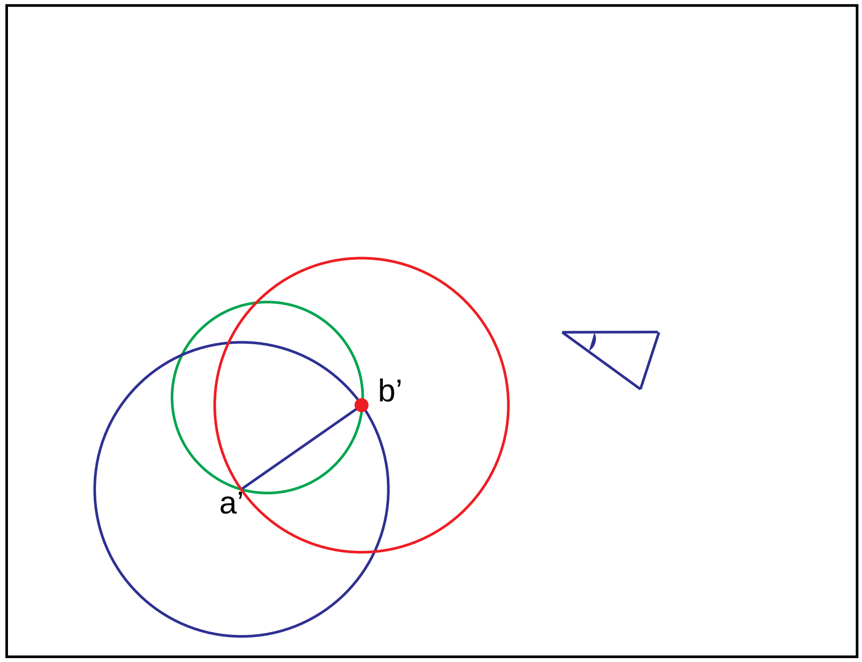
III.1:5. and let it be bisected at the point d’;
([I.10], C#5B)

GOSUB C#5B.

Swing a’b’ around a’.

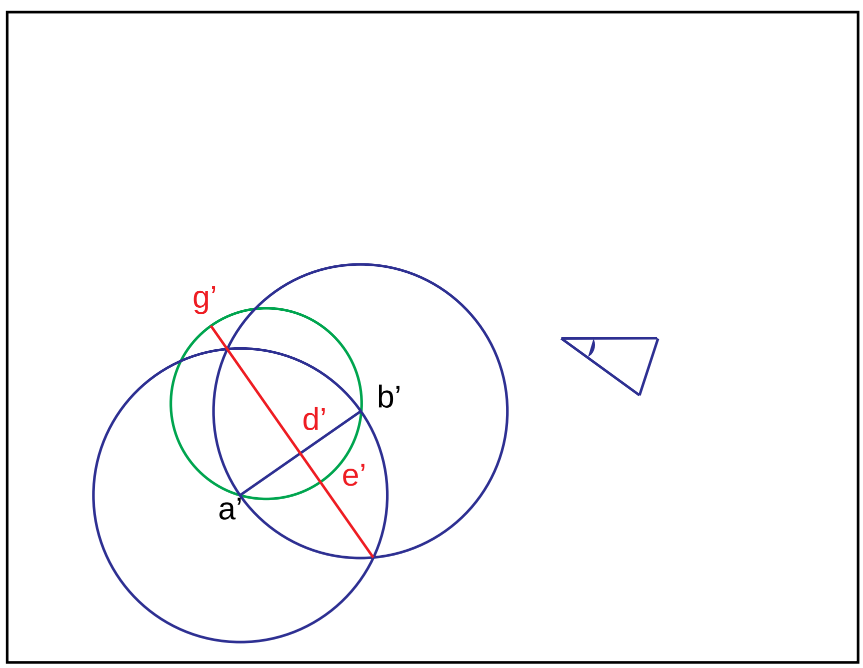


Swing b’a’ around b’.

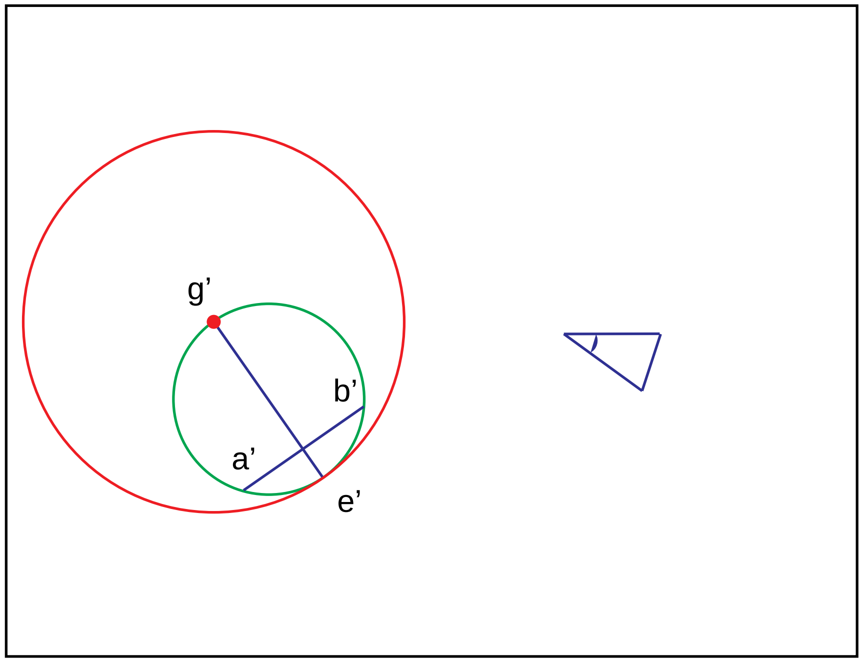


Connect the crssing points.
Extend through the given circle.
Mark d’, e’, g’.

RETURN to III.1:5.
Cleanup.
Retain the line e’g’.

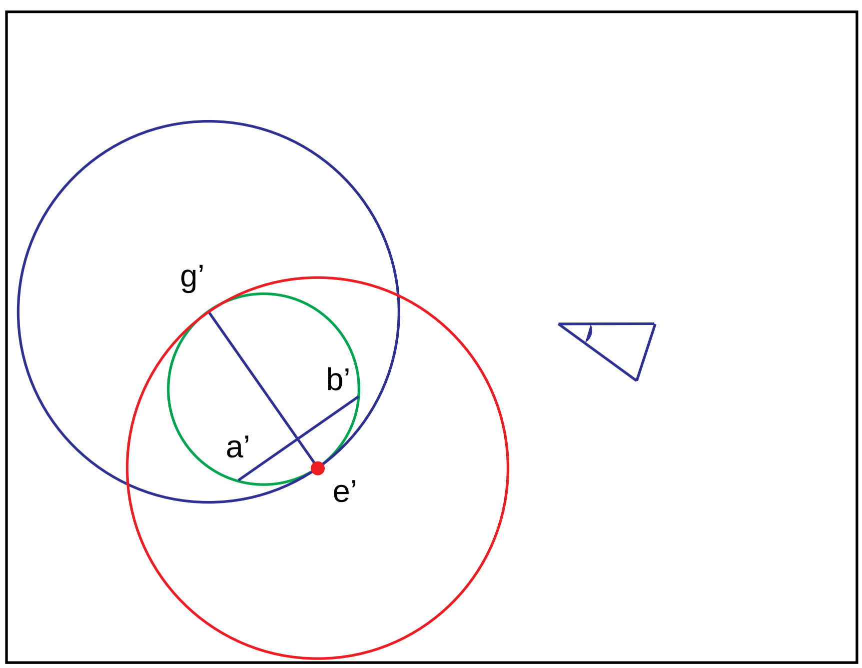


III.1:9. Let g’e’ be bisected at f’.

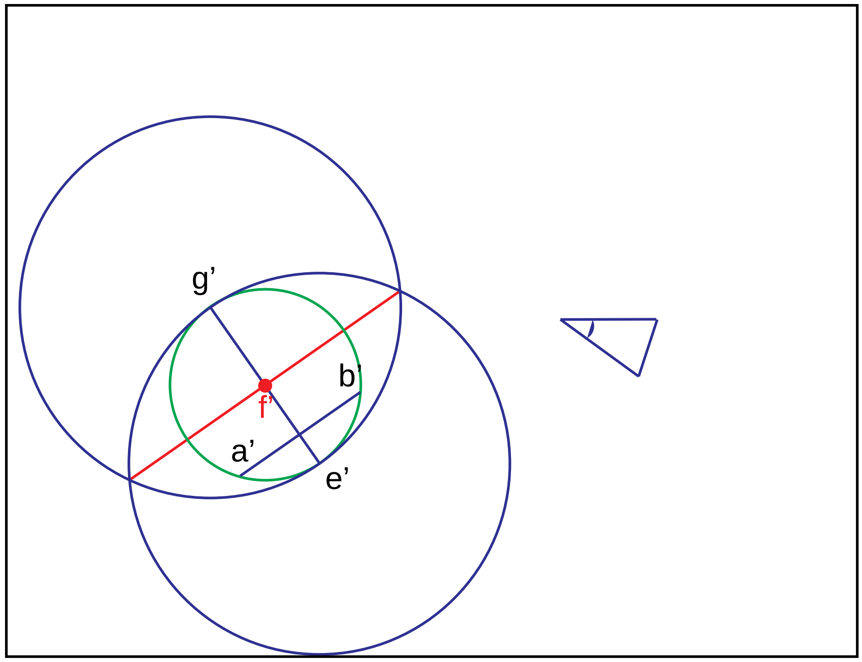


GOSUB C#5B.
Swing g’e’ around g’.

Swing e’g’ around e’.



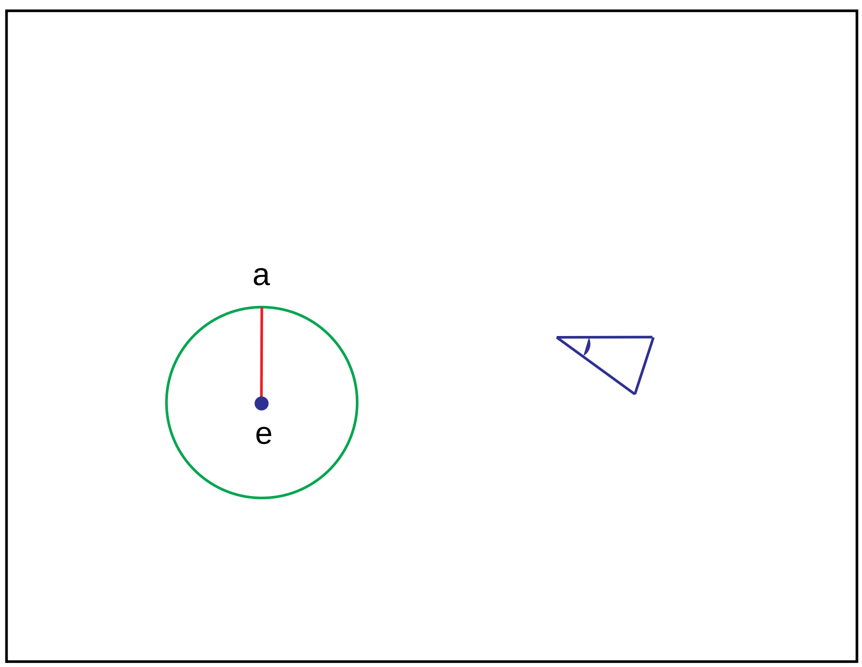
Connect the crossing points.
Mark f'.



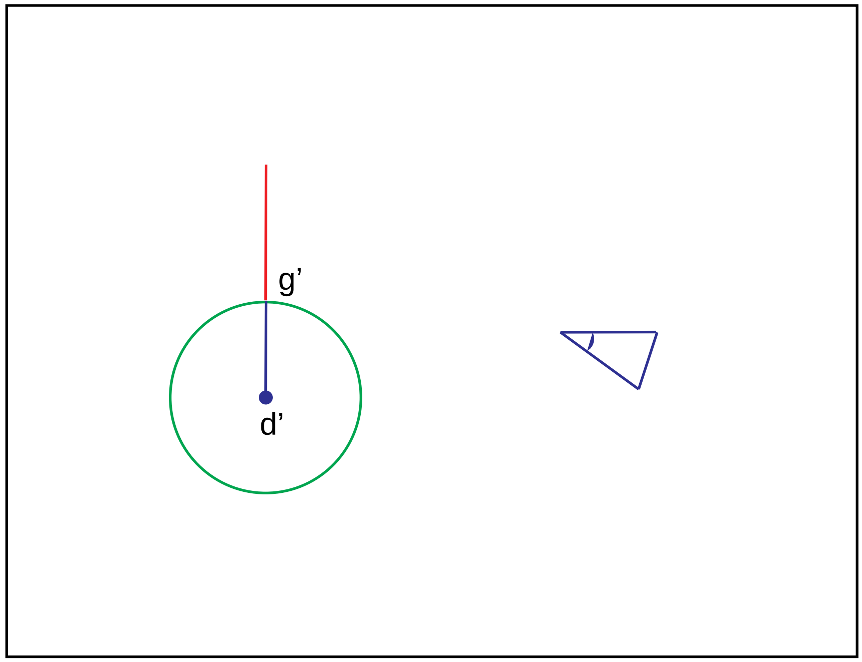
RETURN to III.1:9.
RETURN to III.17:6
Cleanup.
Relabel.

III.17:8. Let ae be joined.

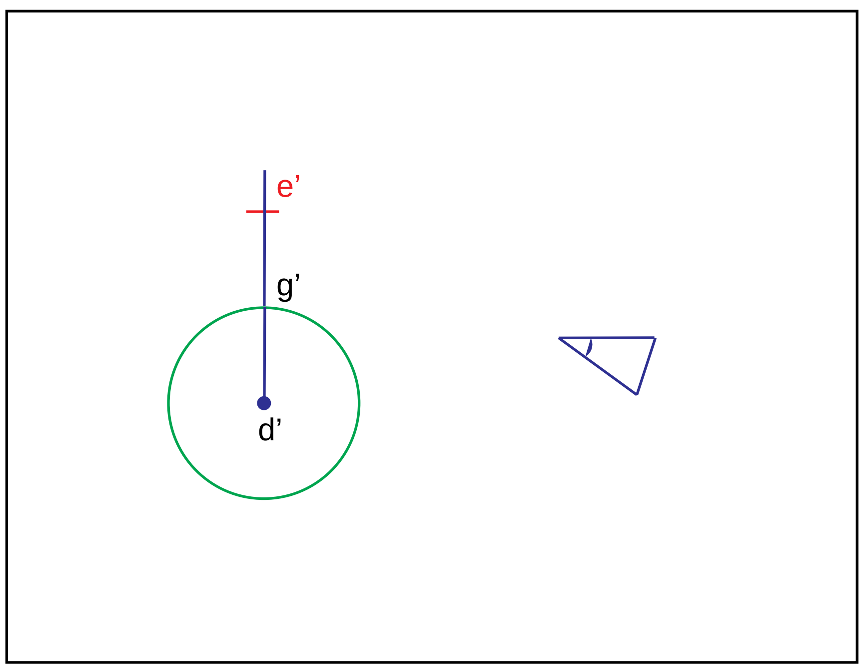
GOSUB I.11



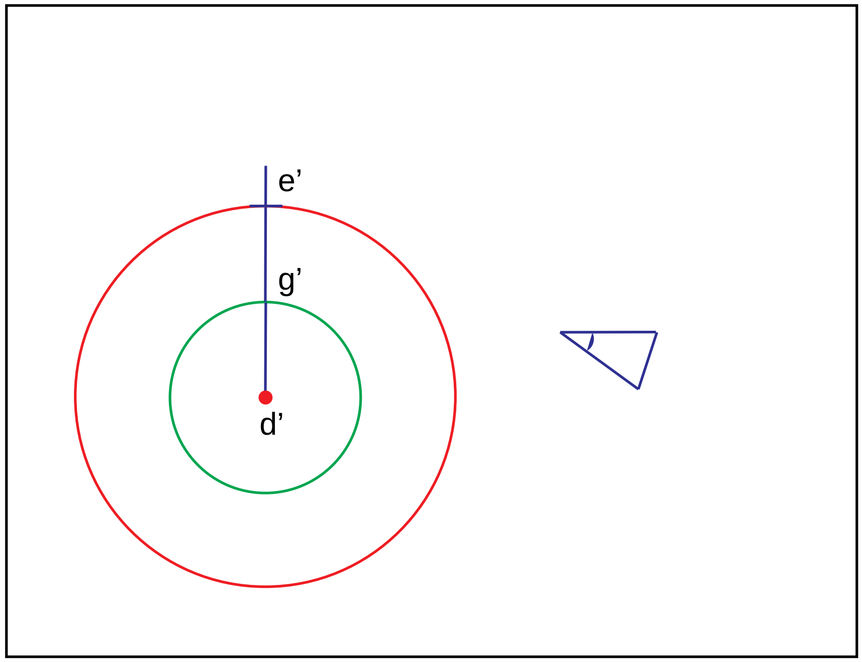
Extend ae.
Relabel.



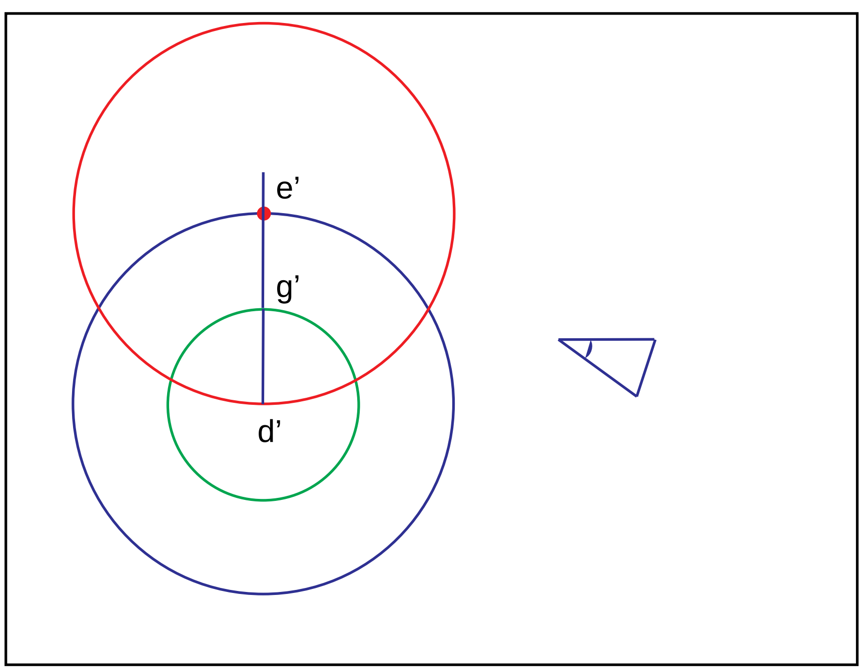
I.11:10. Let g'e' be made equal to g'd'.



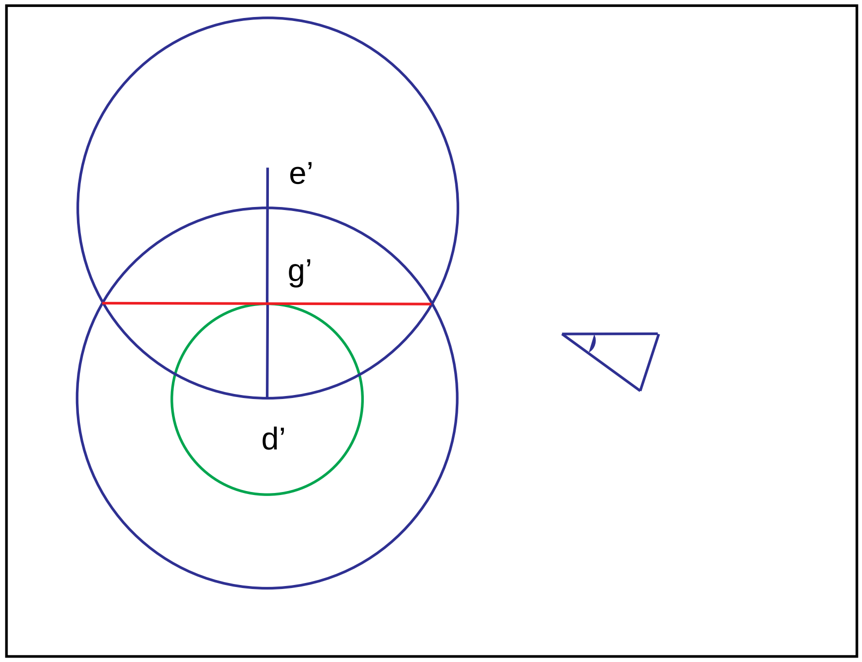
Swing d’e’ around d’.



Swing e’d’ around e’.

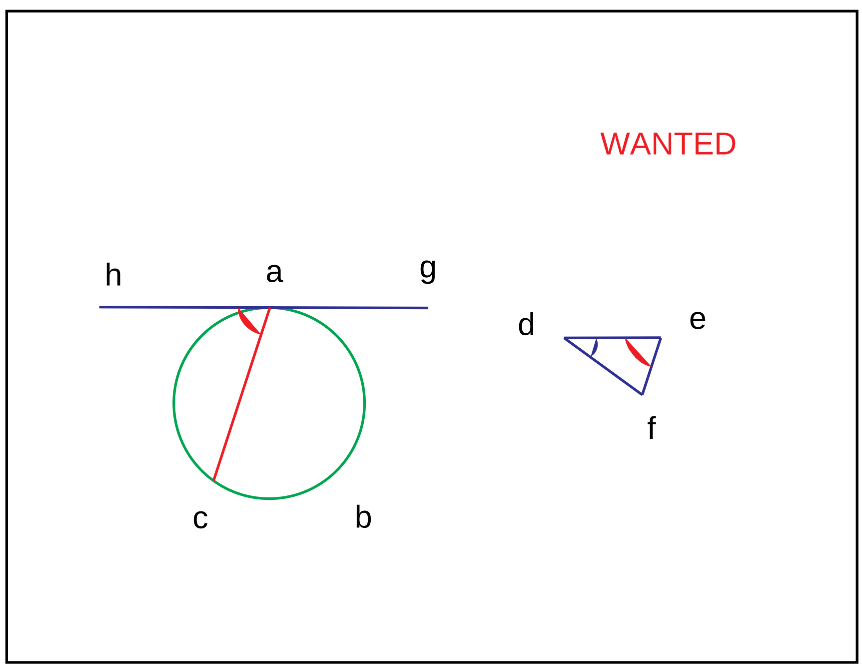


Connect the crossing points.



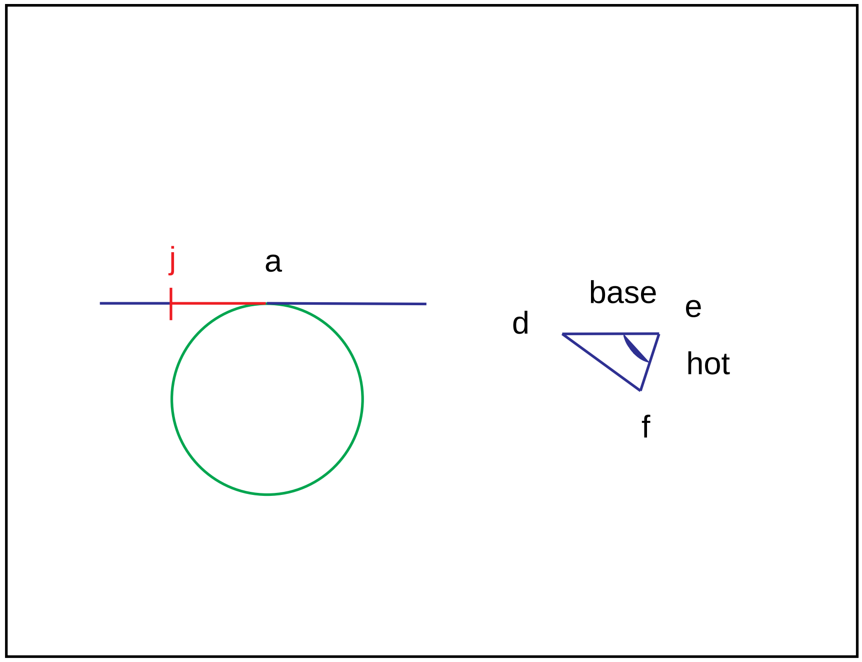
Cleanup.
RETURN to IV.2:7.
Relabel.

IV.2:8. On the straight line ah, and at the point a on it, let the angle hac be constructed equal to the angle def,

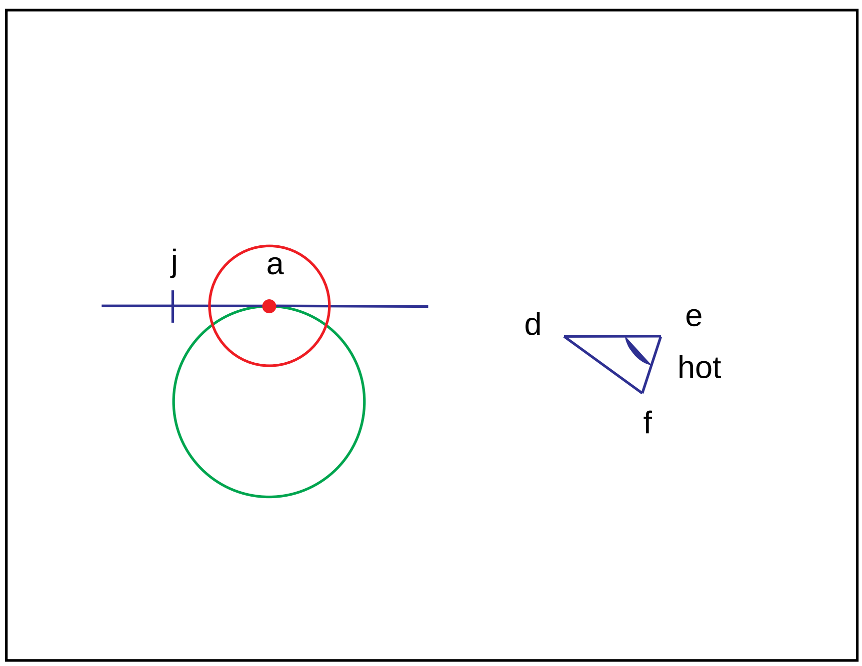


GOSUB C#8P.

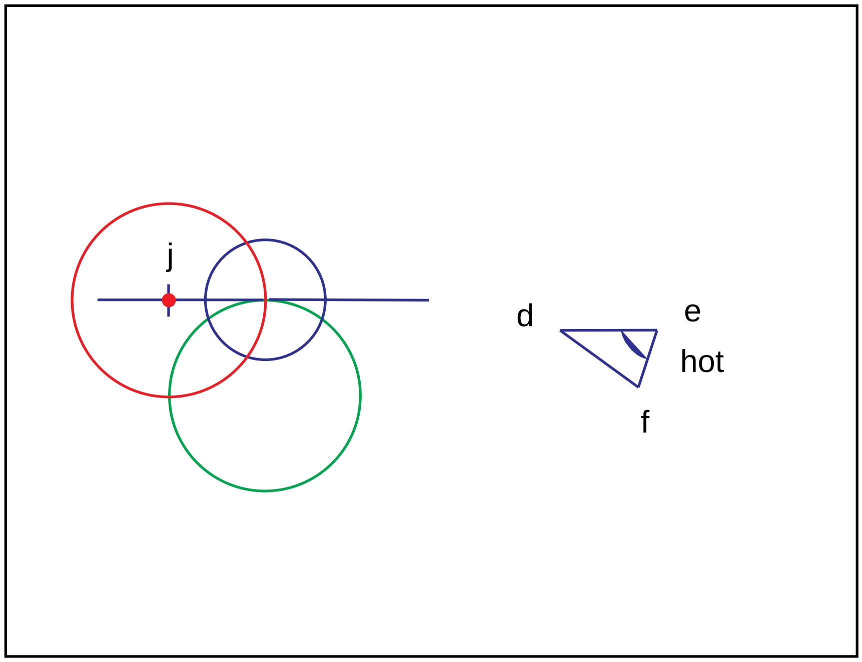
Move the base ed to aj.



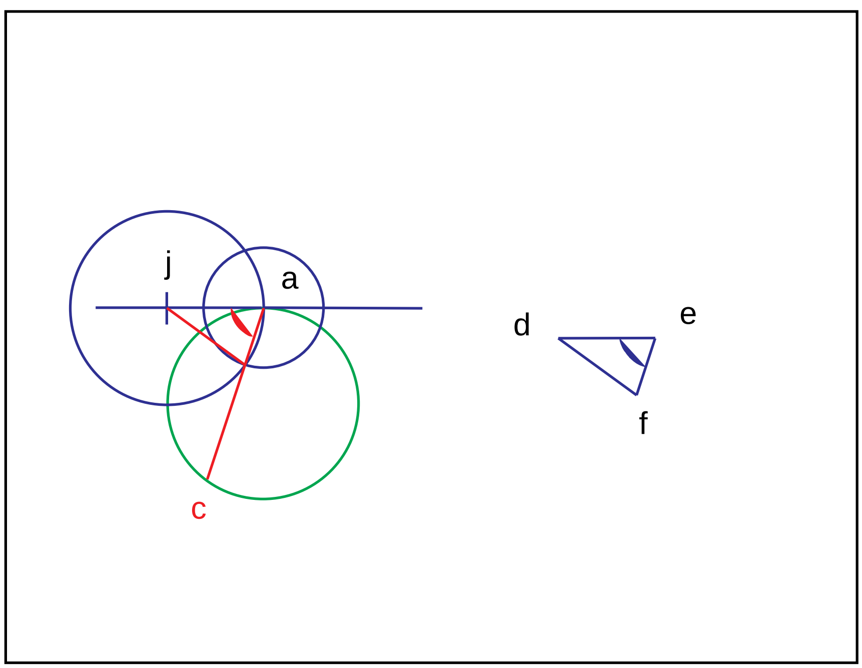
Swing the hot arm ef around a.



Swing the cold arm df around j.



Connect the lower crossing point to both ends of the moved base. Extend the moved hot side across the given circle, locating the point c.

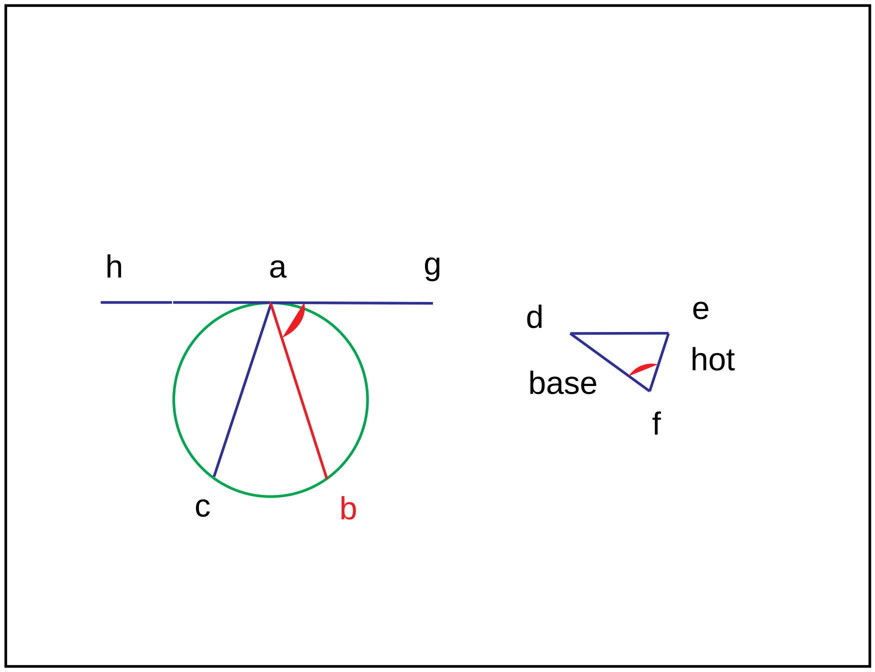


Cleanup.

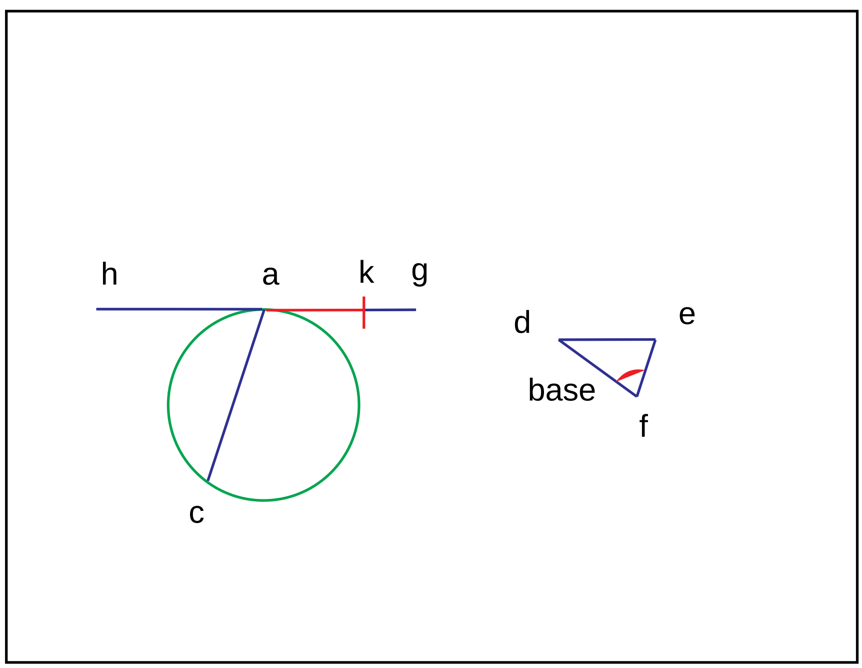
RETURN to IV.2:8.

IV.2:10. and on the line ag, and at the point a on it, let the angle gab be constructed equal to the angle dfe;
[I.23]

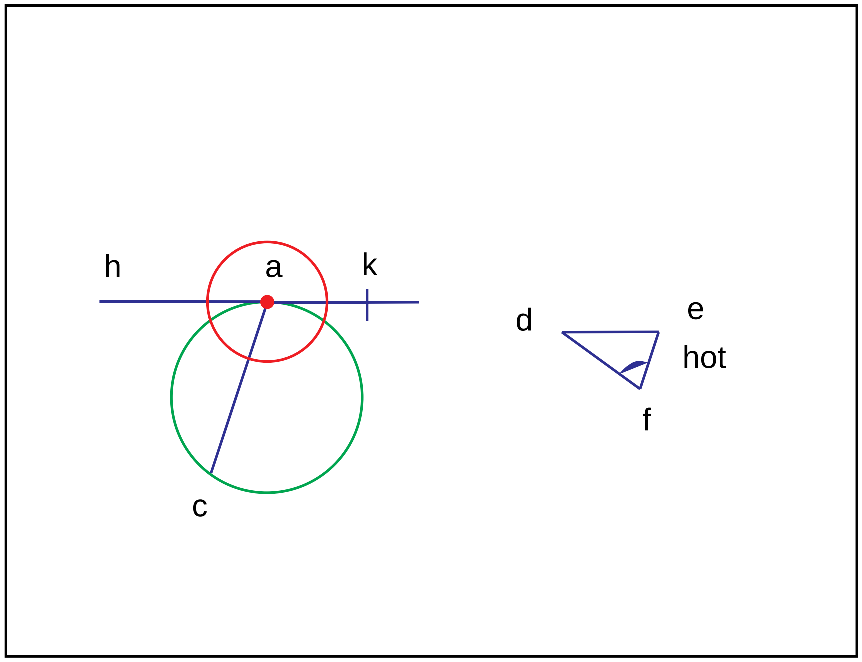
GOSUB C#8P again.



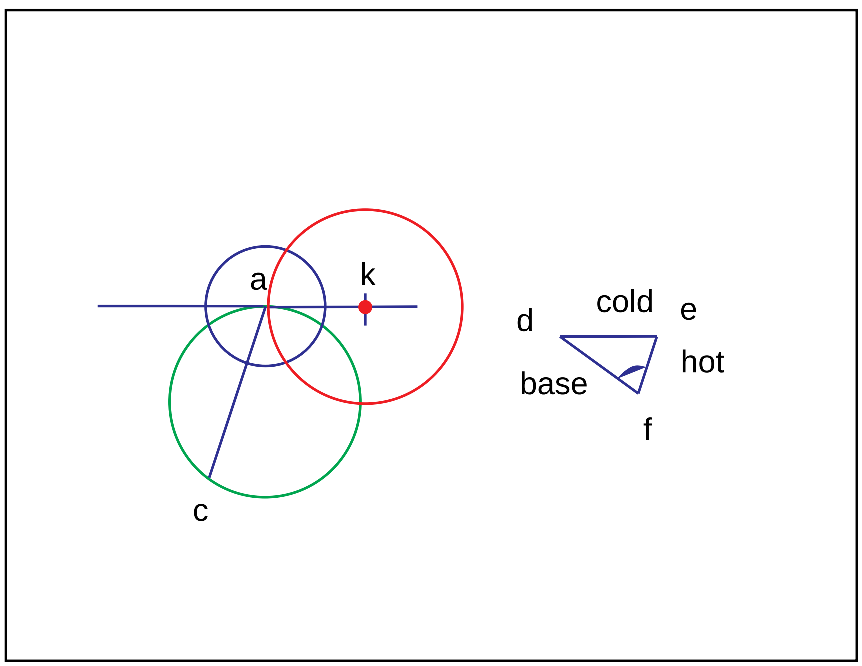
Move the base df to ak along ag.



Swing the hot arm ef around the hot end a of the moved base.

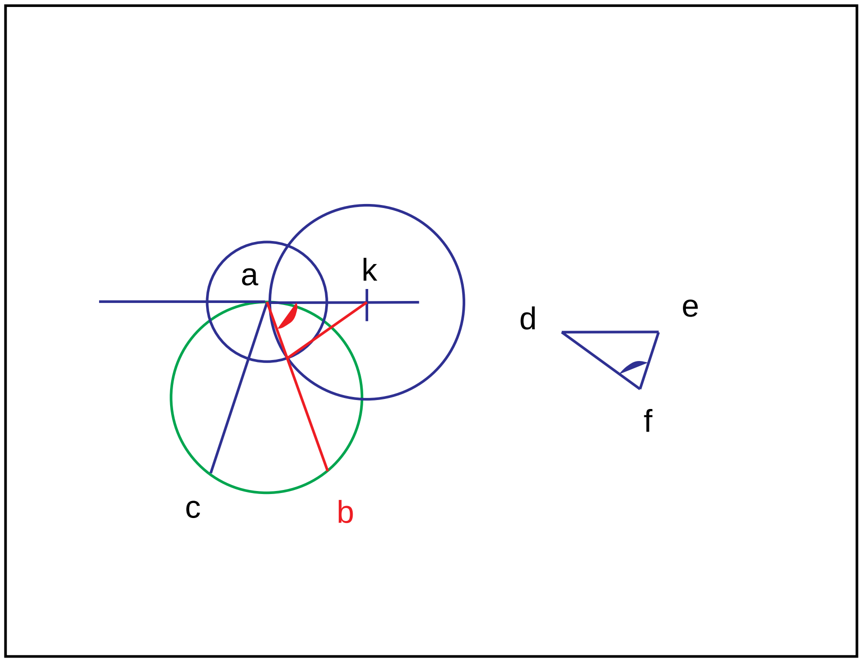


Swing the cold arm df around the cold end k of the moved base.



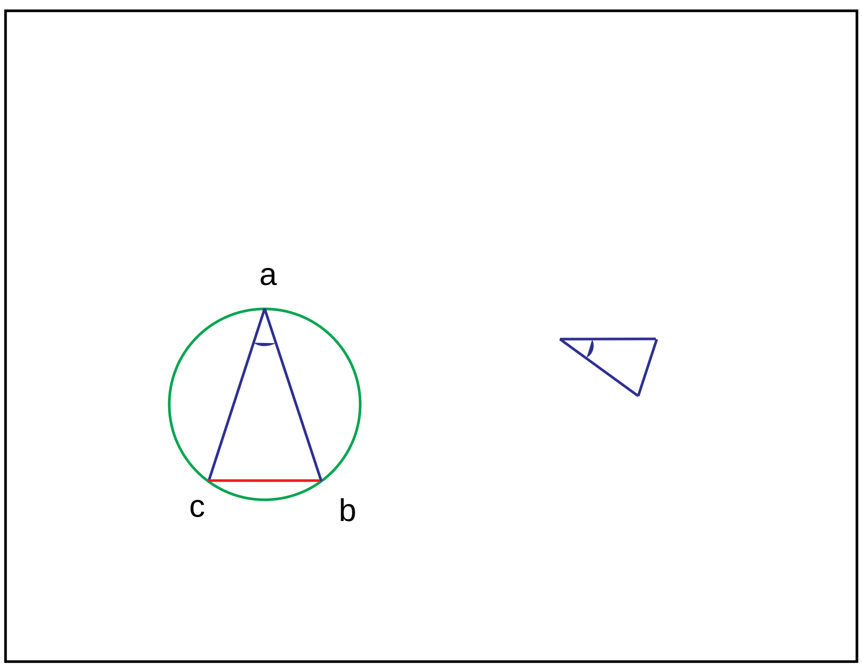
Connect the lower crossing point to both ends of the moved base. Extend the moved hot side across the given circle, locating the point b.

Cleanup.
RETURN to IV.2:10.

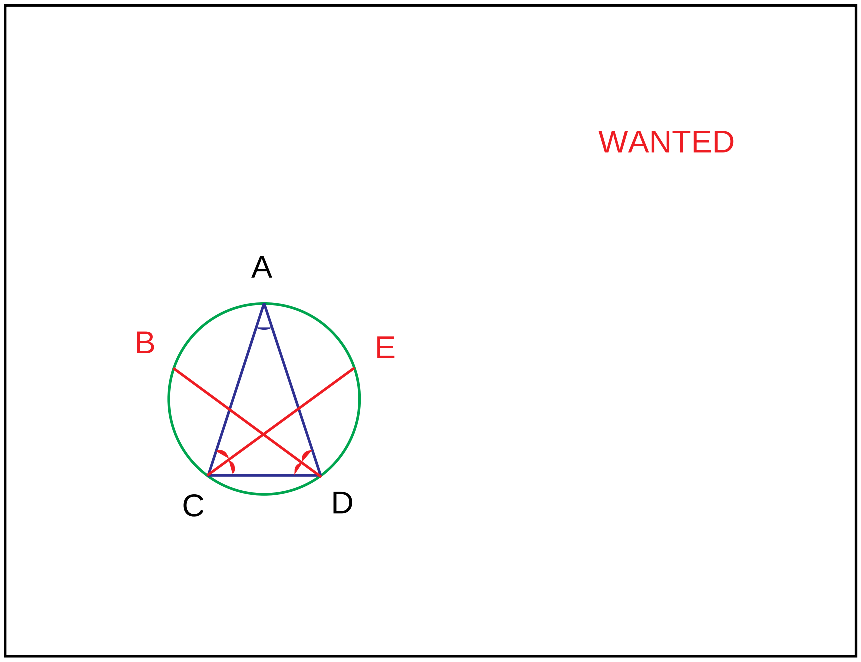


IV.2:12. let bc be joined.

RETURN to IV.11 at line 10.
Relabel.



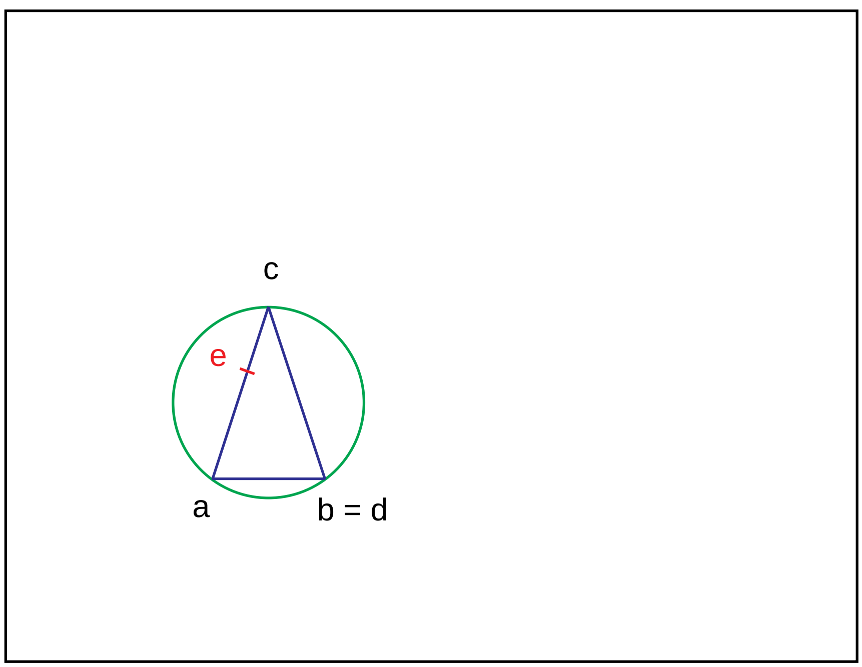
IV.11:17. Now let the angles ACD, CDA be bisected respectively by the straight lines CE, DB [I.9],



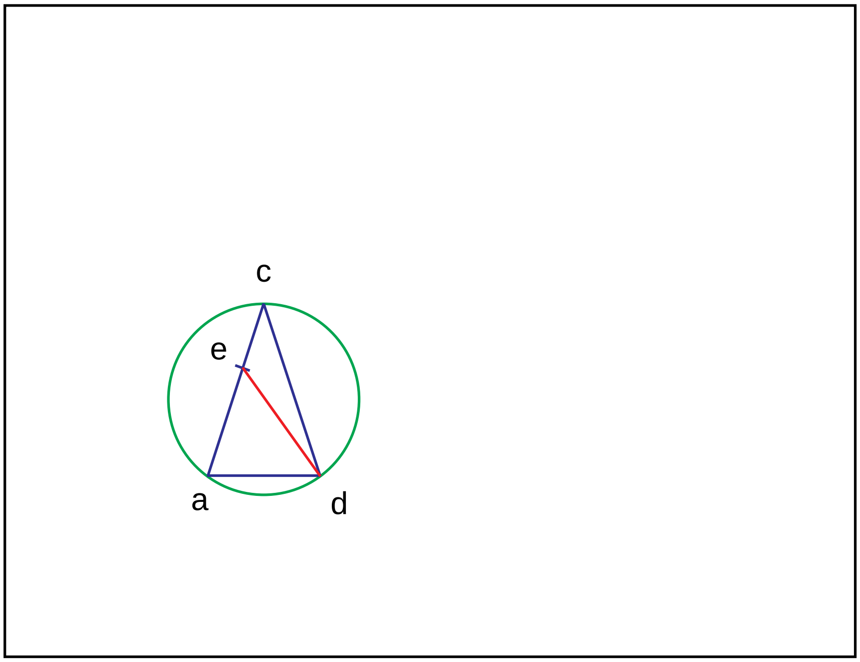
GOSUB I.9 (C#4) for ACD.
Relabel.

I.9:4. Let a point d be taken at random on ab.
We choose d = b.

I.9:5. let ae be cut off from ac equal to ad; [I.3]

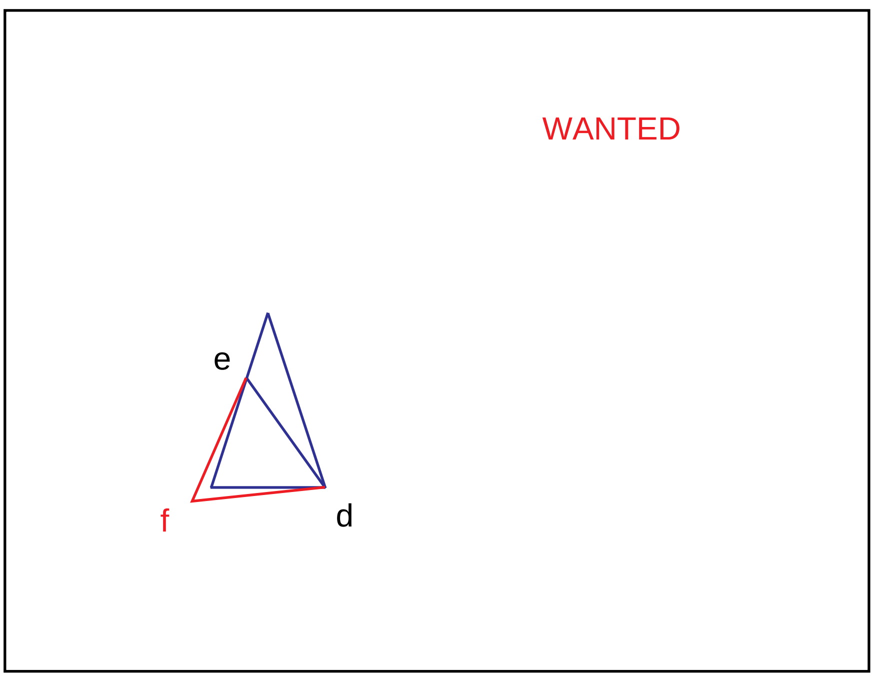


I.9:6. let de be joined,

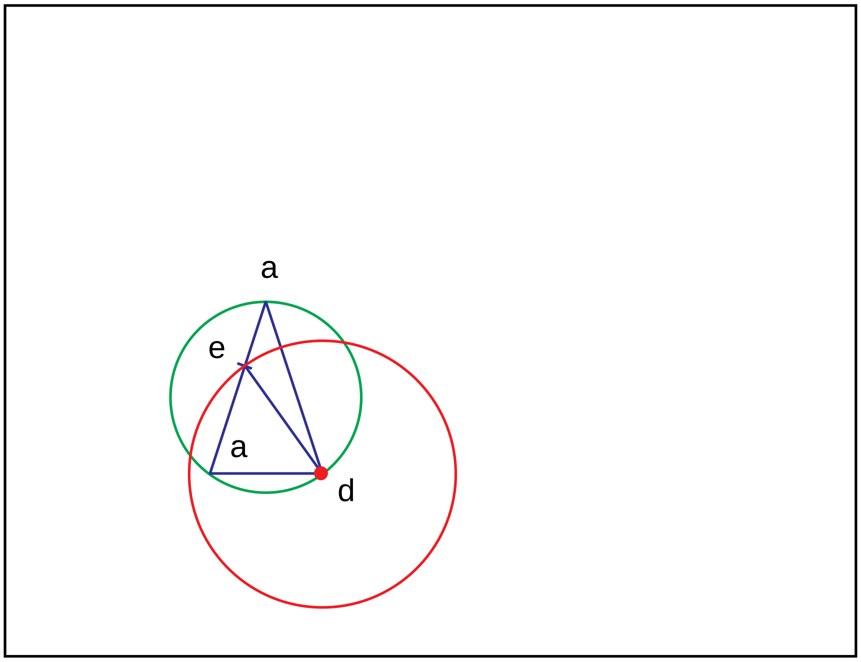


I.9:6. and on de let the equilateral triangle def be constructed; ([I.1])

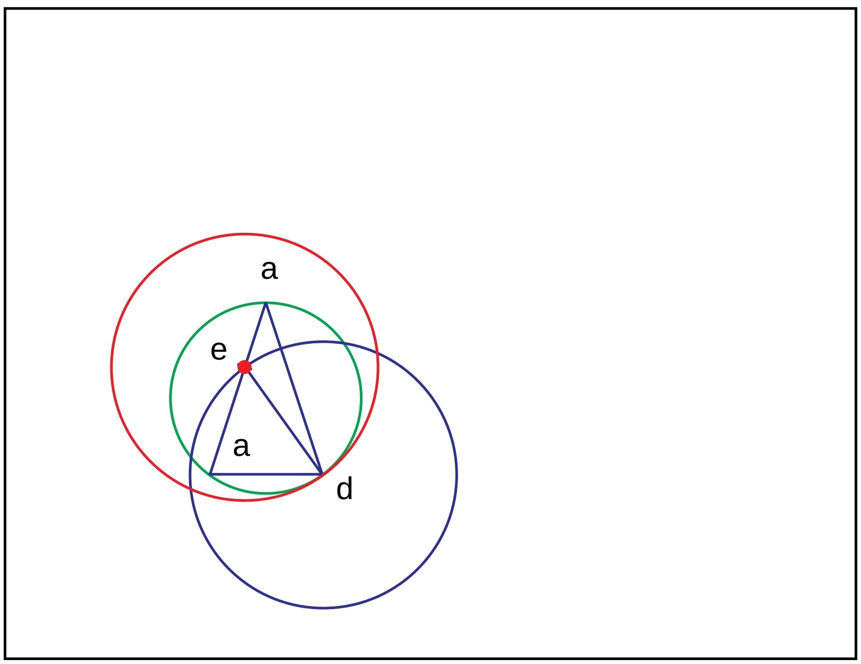
GOSUB I.1 (C#1)



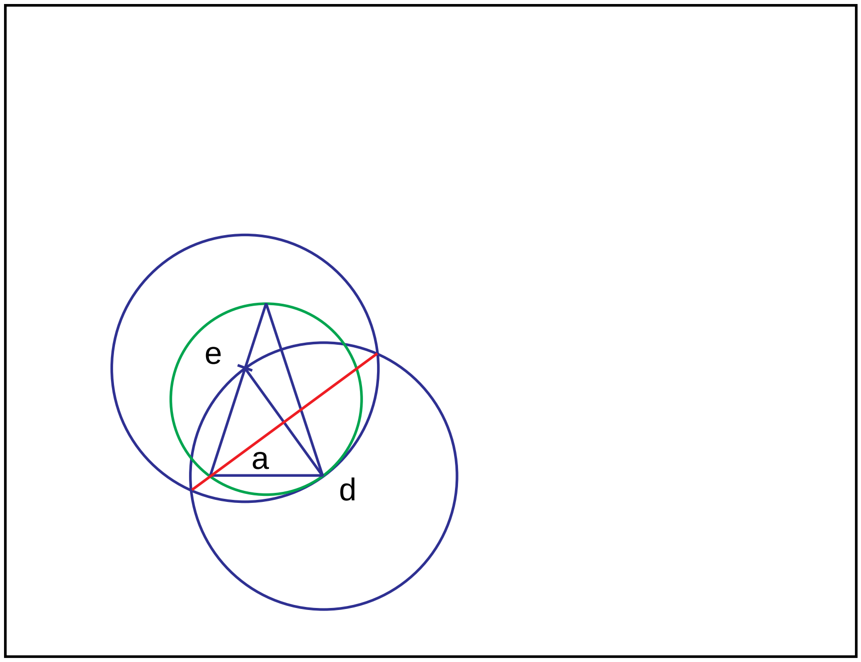
Swing de around d.



Swing ed around e.



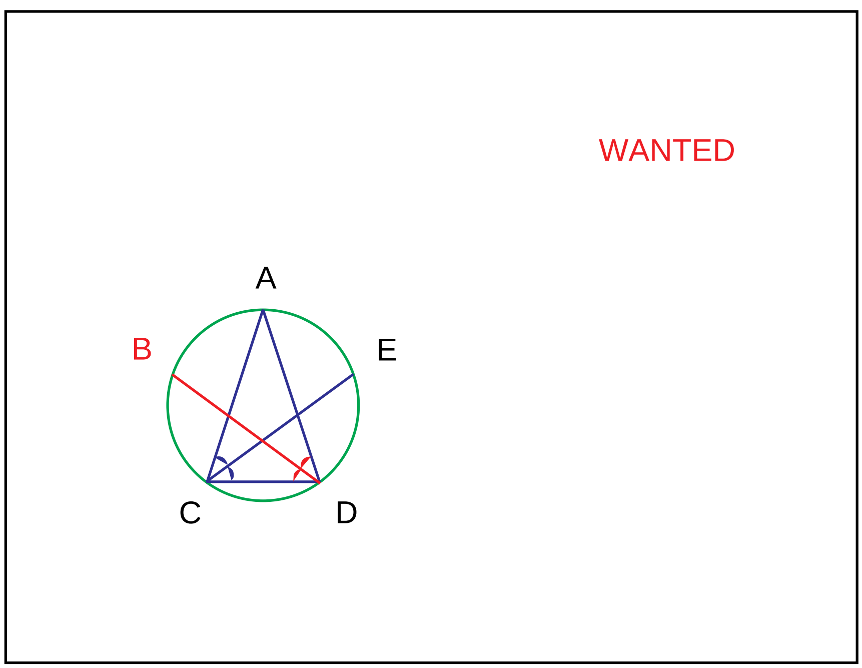
Connect the crossing points.



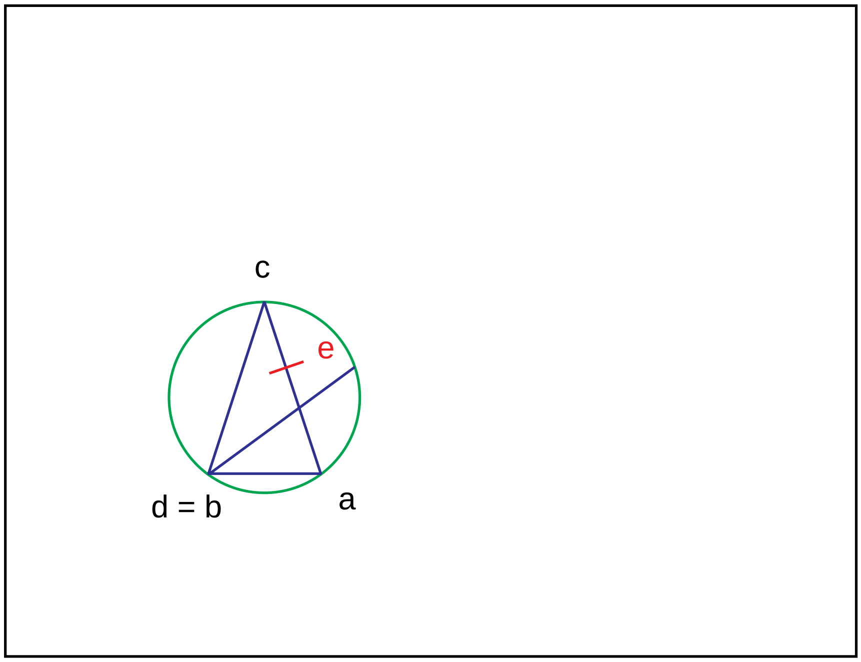
Cleanup.
RETURN to I.9:6.
RETURN to IV.11:17.

(We do not need to finish I.1.)

GOSUB I.9 again for angle CDA.
Relabel.

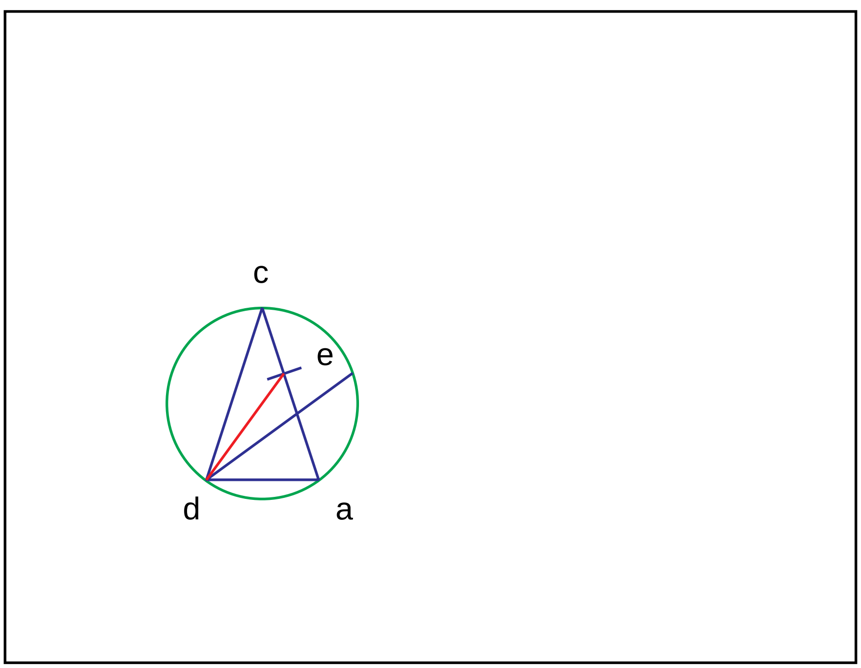


I.9:4. Let a point d be taken at random on ab.
We choose d = b.

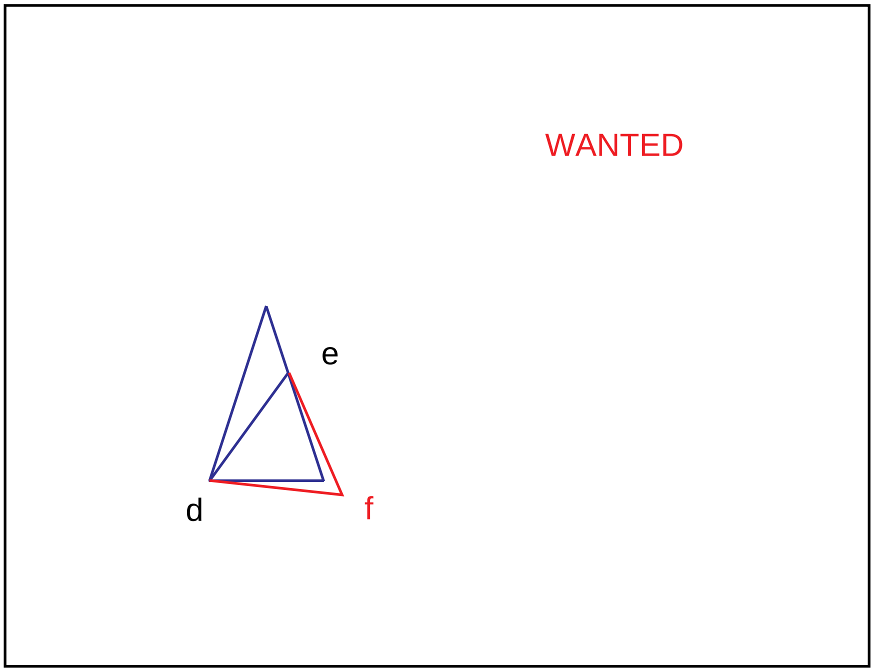


I.9:5. let ae be cut off from ac equal to ad;
[I.3]

I.9:6. let de be joined,

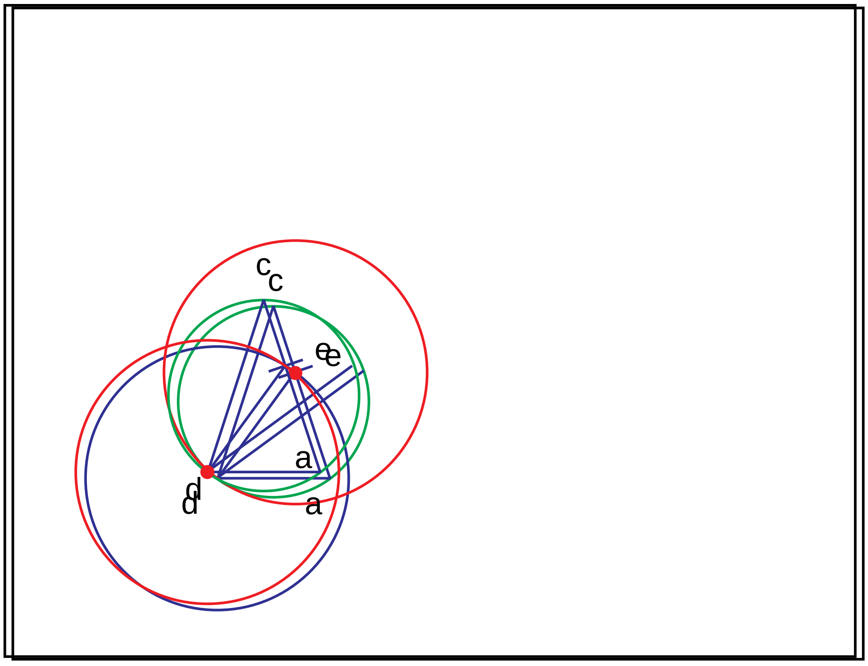


I.9:6. and on de let the equilateral triange def be constructed; ([I.1])

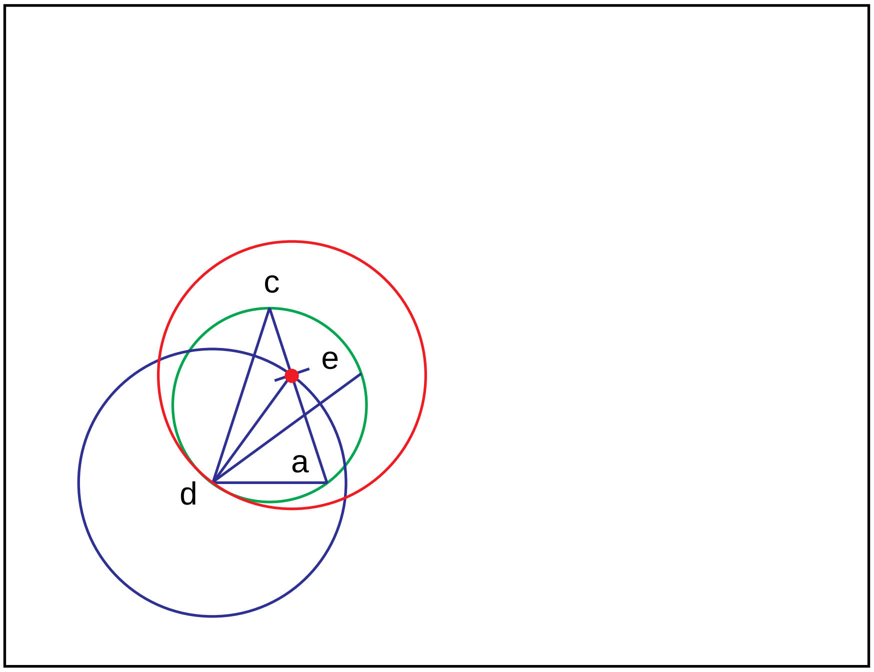


GOSUB I.1.

Swing de around d.

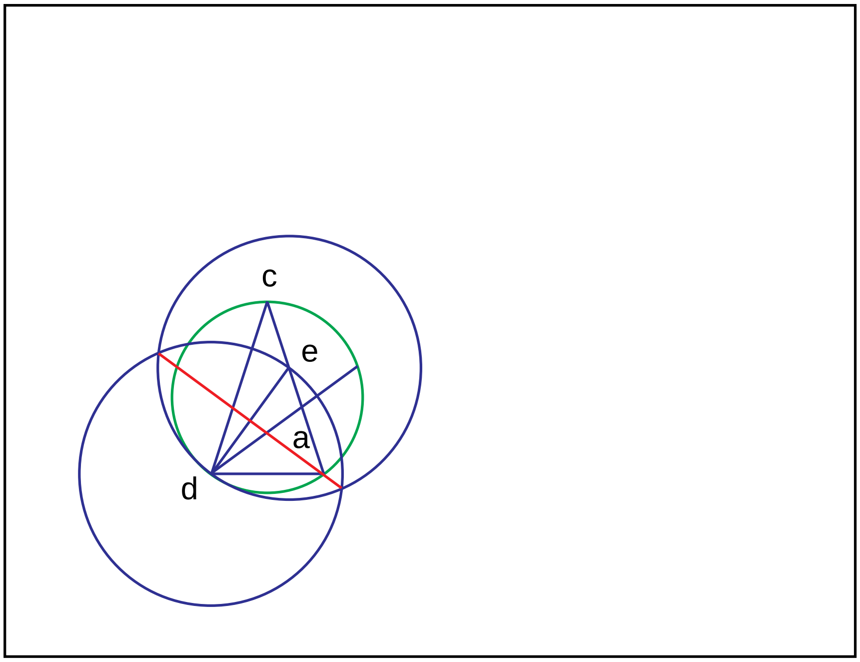


Swing ed around e.

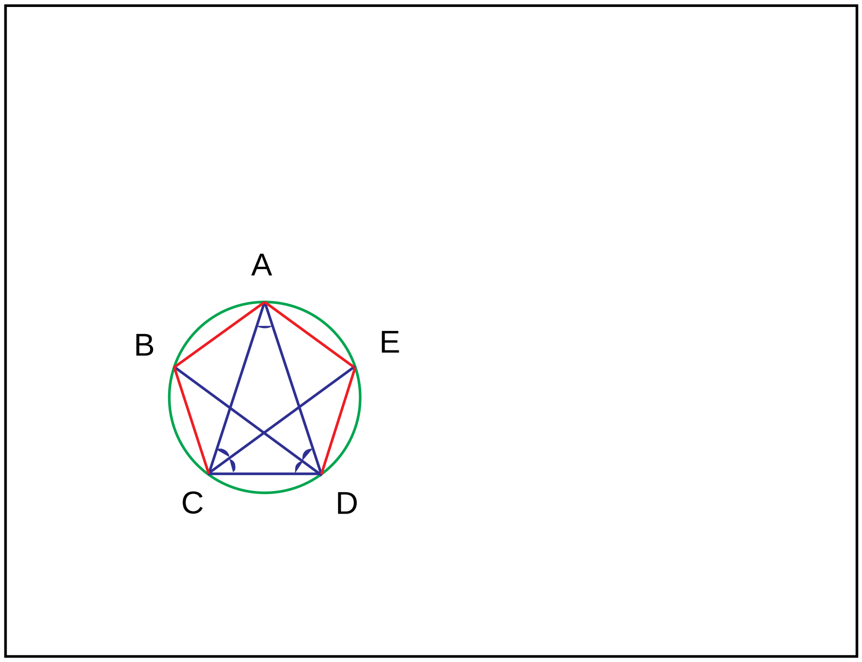


Connect the crossing points.

Cleanup.
RETURN to I.9:6.
RETURN to IV.11:17.
Relabel.

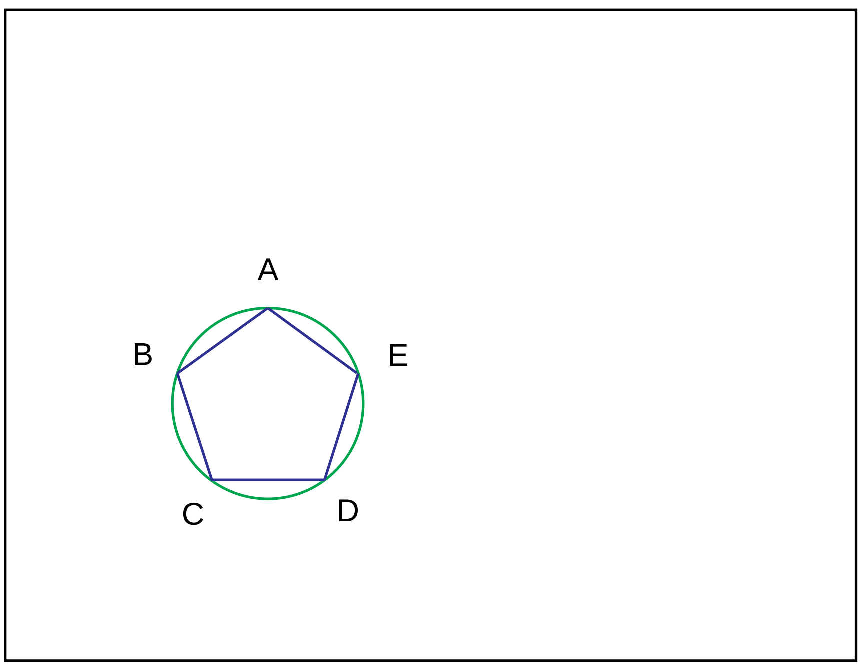


IV.11:18. and let AB, BC, DE, EA be joined.



Cleanup.
We are done.

Q.E.F.



Exercise: Find a 22 step alternative.
Hint: Cf. Heath comment.